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AEROSPACE STANDARD

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Relative Thermal Life and Temperature Index

FOREWORD

Changes in this reaffirm are format/editorial only.

This test method is based upon similar thermal aging tests for insulated wires, developed by IEEE and ASTM D 09.16, but offers important contributions. It adds statistical confidence limits and provides an extensive test example to assist those who have not previously performed this type of long-term thermal testing.

This test method standard was developed to identify a user-friendly test method and provide an aerospace wire user with a means of thermally rating newly developed insulated wires relative to wires that have been used in aerospace vehicles for several decades.

This test method is intended for use in conjunction with other aerospace wire tests. It is not intended as a short-time quality control test procedure. It provides the aerospace vehicle designer and user with a criterion for selection of the most suitable wire for a particular application.

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1. SCOPE:

1.1 This test method provides performance data on candidate insulation systems as a function of time and temperature. These data give engineering information on the wire insulation candidate relative to the performance of materials already in use with a backlog of experience. These tests expose candidate insulation systems to a wide range of temperatures for short and long periods of time, while measuring the degradation of its physical properties. For aerospace use, end-point proof tests include mandrel bend, water soak, and dielectric integrity.

1.2 The thermal rating of an insulation system describes the quantitative relation of life of an insulation system to the temperature and time of exposure. The end of life is defined by the mechanical/ electrical loss of integrity of the insulation system after exposure. A Temperature Index (TI) is then assigned based on temperatures and times recorded during the test. It should be noted that the Index is based upon the time at which 50% of the samples have failed. A related quantity is Relative Temperature Index (RTI) and is obtained when a candidate insulation system is simultaneously tested with a known insulation system using identical conditions (e.g., same temperatures, time intervals, and oven). RTI has been found to be more meaningful than TI since it is less affected by the variations in test conditions and procedures.

1.3 Also included in this document is an extensive appendix to assist the user in performing and interpreting this test. Included in the appendix is an example which provides greater detail for those who have not previously run this test. This type of detail is not included in any of the referenced documents.

1.4 This test method serves several purposes. Initial qualification of new aerospace wires to specifications for both commercial and military users may be established by using the TI and RTI. Also, it provides engineering information data for assigning operating or rating temperatures for a specific insulated wire.

1.5 The high temperature test may be used to statistically compare the thermal performance data of a lot of wire with the original test data. The high temperature test is not recommended for use in continuing production quality control procedures until empirical evidence shows that the high temperature test or another analytical procedure, such as a thermogravemetric test, can accurately provide quality control testing information.

2. APPLICABLE DOCUMENTS:

The following publications form a part of this specification to the extent specified herein. The applicable issue of other publications shall be the issue in effect on the date of the purchase order.

2.1 ASTM Publications:

Available from ASTM, 1916 Race Street, Philadelphia, PA 19103-1187.

ASTM D 3032 Standard Methods of Testing Hookup Wire Insulation - Relative Thermal Life and Temperature Index

2.2 IEEE Publications:

IEEE No. 98 Guide for the Preparation of Test Procedures for the Thermal Evaluation of Electrical Insulating Materials

IEEE No. 101 Guide for the Statistical Analysis of Thermal Life Test Data

3. RELATIVE THERMAL LIFE AND TEMPERATURE INDEX:

3.1 Procedure for Test:

The procedure shall be followed as prescribed in the Relative Thermal Life and Temperature Index Test as defined in ASTM D 3032. Any further references to ASTM D 3032 will refer to the Relative Thermal Life and Temperature Index Test. In the event of conflicts between this document and ASTM D 3032, this document will take precedence. The following items are exceptions or additions to ASTM D 3032.

3.1.1 Sample Selection: The wire specimens shall be made with conductor materials appropriate to the expected operating temperature and use. The results on a finished wire can be affected not only by the insulation materials but also by the conductor materials and platings.

3.1.2 Proof Tests: ASTM D 3032 calls for the following proof-test procedures after each heat aging cycle:

a. Mandrel wrap
b. Water soak
c. Dielectric proof test in water

Where appropriate to the end use, other environmental, mechanical, or electrical proof tests may be used to evaluate the relative thermal life of two or more wires. These other proof tests should only be used to provide relative thermal lives for comparison purposes.

3.1.3 Use of Truncated Data: From 10 to 50% of test time can typically be saved by using truncated data. It is recommended that the Thermal Life Test be discontinued before all ten specimens fail. Discontinue the test when at least six specimens have failed at each test temperature. As in ASTM D 3032, for the use of truncated data, use the log-average time of the fifth and sixth failures as the average life for that temperature. If all ten specimens have failed, use the equation for log-average of ten specimens in ASTM D 3032.

3.1.4 Temperature Selection: Temperature selection shall be according to the conventions of ASTM D 3032. Consideration must be given to temperature dependent physical properties such as melt point when selecting the test temperatures for this procedure.

3.2 Calculations of Results:

It is recommended that at least six significant digits be used in all calculations. Rounding off numbers can significantly affect the results. Calculator or computer packages to perform the regression and confidence interval calculations should also maintain at least six significant digits to assure accuracy. All logarithms used in this test procedure are designated in base 10.

3.2.1 Calculation of Time to Failure: Use ASTM D 3032 with the procedure for calculating individual specimen exposure time to failure and average life of each temperature group as in the calculation of results section.

3.2.2 Arrhenius Plot: Regression analysis of the time/temperature data may be made as follows. For most materials, the time/temperature data follow the Arrhenius relationship which states that the logarithms of the average times to failure have a straight line relationship to the reciprocals of the absolute temperatures to which they are exposed. Table 1 is a model data table of time to failure versus test temperature. This tabular format should be used to record the observed times to failure of a candidate wire construction at the test temperatures used.

3.2.3 Analysis of Test Data: Graphing the resultant data requires a special logarithmic graph paper. The log-average time to failure (hours) is plotted on the ordinate (y axis) versus the reciprocal of the absolute test temperature (1/K) on the abscissa (x axis). The data in Table 1 are used to calculate the values to be plotted and are used to calculate a linear regression line. Theoretically, this plot should be a straight line. This may not be precisely the case. A statistical method known as "least squares" is used to calculate the most probable straight line relationship. The best fit regression line must be determined mathematically with the "least squares" method and plotted on the special logarithmic paper.

TABLE 1 - Model Data Table of Observed Thermal Life Test Data
Time to Failure, in Hours, at Temperatures T_1, T_2, T_3, and T_4

Failure	T_1°C	T_2°C	T_3°C	T_4°C
1	y_{10}=	y_{20} =	y_{30} =	y_{40}=
2	y_{11} =	y_{21} =	y_{31} =	y_{41} =
3	y_{12} =	y_{22} =	y_{32} =	y_{42} =
4	y_{13} =	y_{23} =	y_{33} =	y_{43} =
5	y_{14} =	y_{24} =	y_{34} =	y_{44} =
6	y_{15} =	y_{25} =	y_{35} =	y_{45} =
...				
7	y_{16} =	y_{26} =	y_{36} =	y_{46} =
8	y_{17} =	y_{27} =	y_{37} =	y_{47} =
9	y_{18} =	y_{28} =	y_{38} =	y_{48} =
10	y_{19} =	y_{29} =	y_{39} =	y_{49} =

3.2.3 (Continued):

The dotted line indicates the stopping point of the test when using truncated data. The first index on each data point is the test temperature and the second is the specimen number (with 0 indicating the first specimen) (see Equation 1).

$$y_{14} = \text{the 5th specimen to fail at temperature } T_1 \tag{Eq.1}$$

The log-average time to failure at each test temperature is calculated by summing up the $\log_{10}$ of each time to failure and dividing by the number of test specimens at that temperature as shown in Equation 2. The log-average hours to failure at temperature T_1 may be calculated by use of Equation 3.

$$Y_1 = \frac{\log y_{10} + \log y_{11} + \log y_{12} + \ldots + \log y_{19}}{10} \tag{Eq.2}$$

$$y_1\,(\text{hours}) = \log^{-1} Y \tag{Eq.3}$$

Y_1 = Observed log-average time to failure of all specimens at temperature T_1

This calculation is repeated at each temperature.

3.2.4 Calculation of Regression Line: A regression line may be calculated and then plotted to predict the life of the candidate wire sample at various temperatures. This line has the form

$$\hat{Y}_i = a + bX_i \tag{Eq.4}$$

where:

$\hat{Y}_i$ = The predicted logarithmic life at the selected test temperature (T_i)
X_i = Reciprocal of the absolute temperature (°Kelvin) at the selected test temperature (T_i)

$$X_i = \frac{1}{T_i(°K)} \tag{Eq.5}$$

a = y axis intercept of the calculated regression line
b = Slope of the calculated regression line

3.2.4 (Continued):

Each test temperature used provides a data point for the plot or calculation. The constants a and b are found using a computer program for linear regression or by use of the following formula:

$$a = \overline{Y} - b\overline{X}$$

$$b = \frac{N\Sigma X_i Y_i - (\Sigma X_i)(\Sigma Y_i)}{N\Sigma X_i^2 - (\Sigma X_i)^2} \quad \text{(Eq.6)}$$

where:

N = Number of test temperatures used in the regression plot
$\overline{Y}$ = Average value of Y_i for all test temperatures
$\overline{X}$ = Average value of all X_i for all data points (temperatures)

For N data points (test temperatures)

$$\Sigma X_i = X_1 + X_2 + X_3 + \ldots + X_N$$
$$\Sigma X_i^2 = X_1^2 + X_2^2 + X_3^2 + \ldots + X_N^2 \quad \text{(Eq.7)}$$
$$\Sigma Y_i = Y_1 + Y_2 + Y_3 + \ldots + Y_N$$
$$\Sigma X_i Y_i = X_1Y_1 + X_2Y_2 + X_3Y_3 + \ldots + X_NY_N$$

The calculated regression line, as determined by constants “a” and “b”, may be plotted to indicate the most probable predicted (expected) life versus temperature relationship. The model Arrhenius plot of Figure 6 of ASTM D 3032 indicates the need for calculating the regression line using least squares.

3.2.5 Calculation with Truncated Data: The same calculations may be done using only the first six specimens to fail at each test temperature where the log-average life at each test temperature is the logarithmic average of the fifth and sixth failures. The calculations are identical with those of 3.2.3 except the log-average time to failure at each temperature is calculated per the truncated data paragraph of ASTM D 3032.

3.3 Interpretation of Results:

3.3.1 Temperature Index (TI): The most precise TI, is obtained by calculation from the regression line or less precisely from the graphic plot of the regression line. Typical required life values $(\hat{y}_i)$ range from 10 000 to 100 000 h. The most precise TI value is calculated by substituting the required expected life value into the calculated regression line equation for $\hat{y}_i$ and calculating the TI:

$$\log \hat{y}_i = a + \frac{b}{TI + 273.2} \quad \text{(Eq.8)}$$

or

$$TI = \frac{b}{\log \hat{y}_i - a} - 273.2$$

At $\hat{y}_i$ = 10 000 h, log $\hat{y}_i$ = 4.000

3.3.2 Extrapolation of Temperature Index (TI) from Arrhenius Relationship: The greatest recommended extrapolation from any end data point is 25 °C. If the TI is more than 25 °C less than the lowest temperature data point, an additional test point no greater than 25 °C above the calculated TI needs to be added. This extrapolation limit is required since the slope of the regression line may change at some point on the expected life versus temperature plot. Interpolation on the regression line between any data points is valid provided the data is linear.

3.3.3 Relative Thermal Index (RTI): It is strongly recommended that tests be made simultaneously on a new wire and a wire with known thermal performance since measuring an absolute Temperature Index is subject to many errors, which can be minimized during comparison tests. The RTI can be calculated by the procedure in ASTM D 3032. The relative performance may be displayed by plotting the Arrhenius curves for each wire on the same graph.

3.3.4 Comparison of TI or RTI of this Method to ASTM D 3032: It should be noted that an RTI or a TI figure arrived at by this test procedure should agree with an RTI or TI using the procedure of ASTM D 3032. A quoted TI or RTI figure must be given along with the procedure by which it was determined and the required expected life.

3.4 Confidence Limits (CL):

The predicted logarithmic life value, $\hat{Y}_i$, obtained from the regression analysis is the most probable logarithmic life figure, but provides no indication of the variability of the predicted life at any given temperature. The calculation of 95% confidence limits of the regression line based on the test data provides this information. If an additional ten specimens from the same wire sample are tested at a given temperature, the observed thermal life value from this test will have a 95% probability of fallling between the 95% upper and lower limits as calculated. These confidence limits vary at each test temperature. The confidence limits usually plot elliptically above and below the regression line. The limits are narrowest in the test temperature range and widen as the extrapolation occurs.

3.4.1 Equations for Confidence Limits: To calculate these limits, find the upper and lower confidence limits for values of the logarithm of the predicted life, then calculate the upper and lower confidence limits for predicted life hours from the logarithms. The following formulas are used:

$$\hat{Y}_u = \hat{Y}_i + t(S)(S'_y) \qquad \hat{y}_u = \log^{-1} \hat{Y}_u$$
$$\hat{Y}_L = \hat{Y}_i + t(S)(S'_y) \qquad \hat{y}_L = \log^{-1} \hat{Y}_L \qquad \text{(Eq.9)}$$

$\hat{Y}_u$ = Upper confidence limit of $\hat{Y}_i$ at a given temperature T_i
$\hat{Y}_L$ = Lower confidence limit of $\hat{Y}_i$ at a given temperature T_i
$\hat{y}_u$ = Upper confidence limit of predicted life at a given temperature T_i
$\hat{y}_L$ = Lower confidence limit of predicted life at a given temperature T_i
$\hat{Y}_i$ = Predicted logarithmic life at the selected test temperature T_i
t = Statistic from Appendix A giving Student's distribution for (n-2) degrees of freedom at the desired Confidence Level (CL)
n = Total number of failed test specimens at all test temperatures
CL = Confidence Level (Typically 95%)
S = Pooled estimate of the standard deviation of $Y_{i,j}$ for all failed test specimens at all test temperatures

$$S = \sqrt{\frac{\Sigma(Y_{i,j} - \hat{Y}_i)^2}{n - 2}} \qquad \text{(Eq.10)}$$

$Y_{i,j}$ = The logarithm of the observed time to failure of specimen j at temperature T_i

$$S'_y = \sqrt{\frac{1}{n} + \frac{(X_c - \bar{X})^2}{\Sigma X_i^2 - \frac{(\Sigma X_i)^2}{N}}} \qquad \text{(Eq.11)}$$

S'_y = Intermediate value for calculating confidence limits on the regression line
X_c = The value of X_i at the temperature where confidence limits are being calculated

3.4.1 (Continued):

$\overline{X}$ = Average value of all Xi for all data points (temperatures)

$$\overline{X} = \frac{\Sigma X_i}{N} \quad \text{(Eq.12)}$$

N = The number of test temperatures used in the regression plot

3.4.2 Calculation of Confidence Limits: Calculate $\ddot{Y}_u$ and $\ddot{Y}_L$ at each test temperature. Calculate the confidence limits also at the temperature determined to be the TI to indicate the variability of the TI value. 95% is the standard confidence level used on the predicted values from the regression line.

3.4.3 Plotting of Confidence Limits for Regression Curve: On the graph of the regression line plot, construct the upper and lower confidence limits using dotted lines.

3.4.4 Confidence Limits on Temperature Index (TI): Calculate the confidence limits of the TI temperature using the confidence limits on the predicted log-average life and the calculated regression line:

X_u and X_L are intermediate values used to calculate the statistical spread in the TI.

$$X_u = \frac{X_{TI} + (\hat{Y}_u - \hat{Y}_{TI})}{b}$$

$$X_L = \frac{X_{TI} - (\hat{Y}_u - \hat{Y}_{TI})}{b} \quad \text{(Eq.13)}$$

b = slope of the calculated regression line
X_{TI} = value of X_i at the TI temperature
$\hat{Y}_{TI}$ = predicted value of Y_i on the regression line at the TI temperature
$\hat{Y}_u$ = upper confidence limit of Y_i at a given temperature T_i
$\frac{1}{X_L} - \frac{1}{X_u}$ = statistical spread in the TI at the given confidence level

3.4.4.1 Analysis of TI Confidence Interval Calculations: The confidence interval on the TI is calculated by assuming the confidence lines are parallel to the regression line. Since the confidence lines are actually elliptical, the calculated confidence interval on the TI is only an approximation when Equation 13 is used.

3.4.5 Confidence Limits with Truncated Data: The calculation of confidence limits using truncated data does not meet rigorous statistical requirements to provide an exact mathematical solution. However, the calculation may be considered a reasonable approximation for most applications.

3.5 High Temperature Test:

To verify that a wire sample selected in the future is representative of the wire originally evaluated, the time/temperature conditions for running a high temperature test must be established. For this purpose it is desirable to establish 99% confidence interval limits for observed time to failure at a single test temperature (based on ten specimens). The temperature for this test will be no greater than the highest temperature of the original test data (the temperature will preferably be 130% ± 15% of the previously determined TI). Consideration must be given to temperature dependent physical properties, such as melt point, when selecting the test temperature for this procedure.

3.5.1 Calculations for High Temperature Test: Confidence limits (prediction interval) for an observed time to failure of a wire sample selected in the future are given by:

$$Y_i(99\%) = \hat{Y}_i \pm t(S)(S_y) \qquad \text{(Eq.14)}$$

where:

$Y_i(99\%)$ = 99% confidence limits for observed logarithmic time to failure at test temperature T_i

S_y = Intermediate value for calculating confidence limits on the regression line for a single future sample

$$S_y = \sqrt{1 + S_y'^2} \qquad \text{(Eq.15)}$$

S is the same as Equation 10.

S'_y is the same as Equation 11.

t is from Appendix A, Table A1, last column on the right, for (n-2) degrees of freedom at a 99% confidence level.

3.5.2 Plotting of High Temperature Test Confidence Limits: Calculate the upper and lower 99% confidence limits for the temperature at which the high temperature test was done and also at a temperature 25 °C higher. Assume the confidence limit lines are straight and plot as dotted lines parallel to the regression line.

3.5.3 Range of Expected Life for High Temperature Test: Draw a vertical line across the regression line on the graph at the test temperature. The number of hours indicated where this line intersects the lower 99% confidence limit is the minimum life expected and the number of hours indicated where this line intersects the upper 99% confidence limit is the maximum life expected. The expected life in hours may also be calculated by taking the anti-logarithm of both the upper and lower confidence limits of $Y_i(99\%)$ as follows:

$$Yy_i(99\%) = Log^{-1}Y_i(99\%) \qquad \text{(Eq.16)}$$

where:

$y_i(99\%)$ = 99% confidence limits for observed time to failure at temperature T_i

3.5.3.1 Analysis of Results: The high temperature test analysis assumes the straight regression line originally determined represents the actual performance of the material at the temperature of the high temperature test. This may not be so. If the log-average time to failure of the test specimens fails to fall within the 99% probability limit, either this sample may not be representative of the sample originally tested or the test conditions may not be representative of those used to test the original sample. Review the laboratory test conditions then repeat the test to verify the observed life is not within the 99% confidence limits (see 3.5.3).

3.5.3.2 Test Conditions: The high temperature test on each specimen must duplicate exactly the proof test procedures used on the original sample specimens.

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APPENDIX A
STUDENT T STATISTICS FOR 2 SIDED TESTS

TABLE A1 - Student T Statistics for 2 Sided Tests

t = Student T Statistics for 2 Sided Tests
Values of $t_{n-2,cl}$ for CL = 90, 95, and 99%

n-2	$t_{n-2,90}$	$t_{n-2,95}$	$t_{n-2,99}$
1	6.314	12.710	63.680
2	2.920	4.303	9.925
3	2.353	3.182	5.841
4	2.132	2.776	4.604
5	2.013	2.571	4.032
6	1.943	2.447	3.707
7	1.895	2.363	3.499
8	1.860	2.306	3.355
9	1.833	2.262	3.250
10	1.812	2.228	3.169
11	1.796	2.201	3.106
12	1.782	2.179	3.055
13	1.771	2.160	3.012
14	1.761	2.145	2.977
15	1.753	2.131	2.947
16	1.746	2.120	2.921
17	1.740	2.110	2.898
18	1.734	2.101	2.878
19	1.729	2.093	2.861
20	1.725	2.086	2.845
21	1.721	2.080	2.831
22	1.717	2.074	2.819
23	1.714	2.069	2.807
24	1.711	2.064	2.797
25	1.708	2.060	2.787
26	1.706	2.056	2.779
27	1.703	2.052	2.771
28	1.701	2.048	2.763
29	1.699	2.045	2.756
30	1.697	2.042	2.750
38	1.687	2.025	2.713
40	1.684	2.021	2.704
50	1.676	2.009	2.678
60	1.671	2.000	2.660
80	1.664	1.990	2.639
100	1.660	1.984	2.626
200	1.653	1.972	2.601
500	1.648	1.963	2.586
Infinity	1.645	1.960	2.576

APPENDIX B
SAMPLE CALCULATIONS

B.1 SCOPE:

Sample calculations are shown for the regression lines (Arrhenius plots), for the Temperature Index (TI), for 95% confidence limits, for the high temperature test, and for the related 99% confidence limits. A minimum of six significant figures were used in all statistical calculations as recommended. All logarithms are base 10. (Yielded results may vary slightly due to variances in computer algorithms.)

B.1.1 Test Temperatures:

To illustrate the calculations, a simulated set of data has been synthesized, it is given in Table B1. Ten data points are given at each of four test temperatures (300, 280, 260, and 240 °C). The Temperature Index was first calculated with only the points for 300, 280, and 260 °C. A 10 000 h Temperature Index was calculated to be at about 220 °C, requiring the regression line to be extrapolated by 40 °C (from 260 to 220 °C). Paragraph 3.3.2 states that no more than 25 °C extrapolation from the lowest test point to the TI should be used. Hence, an additional temperature test at 240 °C with ten added specimens is used. Thus, we have four temperature test points.

TABLE B1 - Simulated Observed Thermal Life Test
Data-Hours of Exposure to Failure

Tests at 300, 280, 260, and 240 °C

Sequence of Failure	300 °C (X_1 = 0.00174459)	280 °C (X_2 = 0.00180766)	260 °C (X_3 = 0.00187547)	240 °C (X_4 = 0.00194856)
1	y_{10} = 300 h	y_{20} = 588 h	y_{30} = 1620 h	y_{40} = 3325 h
2	y_{11} = 300 h	y_{21} = 588 h	y_{31} = 1620 h	y_{41} = 3325 h
3	y_{12} = 420 h	y_{22} = 756 h	y_{32} = 1620 h	y_{42} = 3400 h
4	y_{13} = 420 h	y_{23} = 756 h	y_{33} = 1620 h	y_{43} = 3400 h
5	y_{14} = 420 h	y_{24} = 756 h	y_{34} = 1620 h	y_{44} = 3500 h
6	y_{15} = 420 h	y_{25} = 756 h	y_{35} = 1980 h	y_{45} = 3500 h
....	....	....	....	....
7	y_{16} = 420 h	y_{26} = 924 h	y_{36} = 1980 h	y_{46} = 3600 h
8	y_{17} = 420 h	y_{27} = 924 h	y_{37} = 1980 h	y_{47} = 3600 h
9	y_{18} = 540 h	y_{28} = 924 h	y_{38} = 1980 h	y_{48} = 3675 h
10	y_{19} = 540 h	y_{29} = 924 h	y_{39} = 1980 h	y_{49} = 3675 h

NOTE: The test point at 240 °C was not used initially but was added due to the 25 °C extrapolation criteria. The dotted line indicates the stopping point of the test when using truncated data.

B.1.2 Calculation of Y_i at Each Temperature:

At each test temperature, the log-average life is calculated using Equation B1 and the results are summarized in Table B2.

At 300 °C, log-average life is calculated as:

$$Y_1 = \frac{2\log 300 + 6\log 420 + 2\log 540}{10} = 2.61585 \quad \text{(Eq.B1)}$$

$$y_1 = \log^{-1} 2.61585 = 413\,h$$

similarly, at 280 °C

$$Y_2 = \frac{2\log 588 + 4\log 756 + 4\log 924}{10} = 2.89155$$

$$y_2 = 779\,h$$

similarly, at 260 °C

$$Y_3 = \frac{5\log 1620 + 5\log 1980}{10} = 3.25309$$

$$y_3 = 1791h$$

for additional test point at 240 °C

$$Y_4 = \frac{2\log 3325 + 2\log 3400 + 2\log 3500 + 2\log 3600 + 2\log 3675}{10}$$

$$Y_4 = 3.54378$$

$$y_4 = 3498\,h$$

TABLE B2 - Observed Time to Failure

T_i (°C)	X_i (1/°K)	(All 10 Specimens) y_i (h)	(All 10 Specimens) Y_i	(Truncated Data) y_i (h)	(Truncated Data) Y_i
300	0.00174459	413	2.61585	420	2.62325
280	0.00180766	779	2.89155	756	2.87852
260	0.00187547	1791	3.25309	1791	3.25309
240	0.00194856	3498	3.54378	3500	3.54407

B.1.3 Calculation of X_i:

Using Equation B2, one calculates X_i at all test temperatures:

$$\text{At } T_i = 300\,°C \quad X_i = \frac{1}{T_i + 273.2} = \frac{1}{300 + 273.2} = 0.00174459 \qquad \text{(Eq.B2)}$$

similarly for T_i = 280, 260, and 240 °C.

B.1.4 Truncated Data:

If, as recommended, tests are disconnected after only six of the ten test specimens have failed, the data are referred to as “truncated data” and the log-average is calculated by Equations 2 and 3 (see 3.2.5): For T_i = 300 °C.

$$Y_1 = \frac{\log 420 + \log 420}{2} = 2.62325 \qquad \text{(Eq.B3)}$$

$$y_1 = \log^{-1}(2.62325) = 420$$

And similarly for T_i = 280, 260, and 240 °C. The calculations are tabulated in Table B2.

B.1.5 Arrhenius Plot:

The Arrhenius plot now can be made in the cases of complete or truncated data using special graph paper as shown in Figure 6 of ASTM D 3032. A regression line calculated by the least squares method should be drawn on the plot (see B.2.1).

B.1.6 Analysis of Results:

For many purposes the three or four temperature points using either complete or truncated data will provide an adequate TI. However, the TI alone provides no numerical measure of the precision unless statistical analysis is performed. Section B.2 illustrates this analysis.

B.1.7 Temperature Index:

In addition to presenting the thermal performance by the graph above, a compact, but less informative representation is the Temperature Index. Select the required life (number of hours) and read the corresponding temperature. The TI or Temperature Index is expressed as:

a. TI - Number of Hours in 1000s/Temperature °C

B.1.8 Notation:

As an example, for 10 000 h (10 kh), where the corresponding temperature on the Arrhenius line is 220 °C, the notation is:

b. TI 10 kh/220

B.2 STATISTICAL ANALYSIS OF DATA:

Find the straight line that is the closest fit to the four test temperature log-average life points. This procedure is known as "least squares."

B.2.1 Least Squares Calculation:

The equations to be used are:

$$a = \overline{Y} - b\overline{X} \quad \textbf{(Eq.B4)}$$

$$b = \frac{N\Sigma X_i Y_i - (\Sigma X_i)(\Sigma Y_i)}{N\Sigma X_i^2 - (\Sigma X_i)^2}$$

The simulated data used are values of X_i and Y_i from Table B2. For illustration, we use the complete data (all ten specimens completed life test).

B.2.1 (Continued):

ΣX_i = 0.00174459 + 0.00180766 + 0.00187547 + 0.00194856 = 0.00737628
ΣX_i^2 = $(0.00174459)^2 + (0.00180766)^2 + (0.00187547)^2 + (0.00194856)^2$ = 0.0000136255
ΣY_i = 2.61585 + 2.89155 + 3.25309 + 3.54378 = 12.30427
$\Sigma X_i Y_i$ = (0.00174459 x 2.61585) + (0.00180766 x 2.89155) + (0.00187547 x 3.25309) + (0.00194856 x 3.54378) = 0.0227969
N = 4 Test Temperatures (Data Points)

These calculated values may be substituted into Equation B4 to find b, the slope estimator. The "Y" intercept may be calculated using Equation B4, where $\bar{Y}$ = average value of Y_i for all test temperatures, and $\bar{X}$ = average value of all X_i for all data points. It is preferred to use a linear regression program to calculate a and b in order to avoid arithmetic or round-off errors. The following slope and intercept estimators were calculated from a linear regression program:

a = -5.45055
b = 4623.81

Similarly truncated data could be used yielding:

a = -5.43616
b = 4615.32

The values of a and b above yield the following regression equations to calculate expected life:

$$Y = -5.45055 + 4623.81(X) \text{ (10 data points)} \qquad \text{(Eq.B5)}$$
$$Y = 5.43616 + 4615.32(X) \text{ (truncated data)}$$

B.2.2 Comparison of Observed and Predicted Values:

The observed times to failure may now be compared to the predicted values using the regression equation calculated in B.2.1. Substituting the several values of "X" in the regression equation gives predicted values of log-average life which are close to, but not identical with, the values of observed log-average life from the test data (see Table B3).

This example shows accuracy between truncated data and ten data points to be very good. Accuracy may not always be this case but is generally very good.

TABLE B3 - Log-Average Life

Test Temperature	(10 Data Points) Observed Life y_i	(10 Data Points) Predicted Life y_i	(Truncated Data) Observed Life y_i	(Truncated Data) Predicted Life y_i
300 °C	413 h	413.1 h	420 h	412.7 h
280 °C	779 h	808.6 h	756 h	806.8 h
260 °C	1791 h	1664.4 h	1791 h	1658.6 h
240 °C	3498 h	3624.2 h	3500 h	3606.4 h

B.2.3 TI:

Use Equation B6 to calculate the appropriate temperature for a TI (Temperature Index) at the given expected life. The following calculation used the complete data (all ten specimens).

$$TI = \frac{b}{\log \hat{y}_i - a} - 273.2 \quad \text{(Eq.B6)}$$

$$TI = \frac{4623.81}{\log 10\,000 + 5.45055} - 273.2 = 216\ °C$$

B.3 CONFIDENCE LIMITS:

Confidence limits may be calculated for the mean life of an additional ten specimens tested at a temperature within the range of the test data. This is expressed as a probability that the mean life will fall within the confidence limits calculated. Usually 95% probability is used.

B.3.1 Calculation of Confidence Limits:

The confidence limits on the log-average times to failure are calculated by Equation B7 using the factors given in Equations B8 and B9.

$$\hat{Y}_u = \hat{Y}_i + t(S)(S_y^{'}) \qquad \hat{y}_u = \log^{-1} \hat{Y}_u$$
$$\hat{Y}_L = \hat{Y}_i - t(S)(S_y^{'}) \qquad \hat{y}_L = \log^{-1} \hat{Y}_L \quad \text{(Eq.B7)}$$

$$S = \sqrt{\frac{\Sigma(Y_{i,j} - \hat{Y}_i)^2}{n-2}} \quad \text{(Eq.B8)}$$

B.3.1 (Continued):

$$S_y^{'} = \sqrt{\frac{1}{n} + \frac{(X_c - \bar{X})^2}{\Sigma X_i^2 - \frac{(\Sigma Xi)^2}{N}}} \qquad \text{(Eq.B9)}$$

The calculation of the pooled standard deviation involves all 40 observed test points.

For 300 °C:

X_i = 0.00174459
$\hat{Y}_i$ = Log of predicted value from regression line at 300 °C
$\hat{Y}_i$ = a + bX_i = -5.45055 + 4623.81 (0.00174459) = 2.61609

Duplicate the calculations shown above for 300 °C at 280, 260, and 240 °C. The results are obtained in Table B4.

TABLE B4 - Predicted Life

Test Temperature	Y_i Logarithmic Predicted Life	y_i Predicted Life (h)
300 °C	2.61609	413.1
280 °C	2.90771	808.6
260 °C	3.22126	1664.4
240 °C	3.55921	3624.2

Use Equation B10 to calculate the pooled standard deviation estimate.

$$S = \sqrt{\frac{\Sigma(Y_{i,j} - \hat{Y}_i)^2}{n - 2}} \qquad \text{(Eq.B10)}$$

B.3.1 (Continued):

where:

$Y_{i,j}$ = Logarithm of the observed time to failure of each specimen at 300 °C

$$S = \sqrt{\frac{\begin{array}{l} 2(\log 300 - 2.61609)^2 + 6(\log 420 - 2.61609)^2 + 2(\log 540 - 2.61609)^2 \\ +2(\log 588 - 2.90771)^2 + 4(\log 756 - 2.90771)^2 + 4(\log 924 - 2.90771)^2 \\ +5(\log 1620 - 3.22126)^2 + 5(\log 1980 - 3.22126)^2 \\ +2(\log 3325 - 3.55921)^2 + 2(\log 3400 - 3.55921)^2 \\ +2(\log 3500 - 3.55921)^2 + 2(\log 3600 - 3.55921)^2 + 2(\log 3675 - 3.55921)^2 \end{array}}{(40-2)}}$$

S = 0.0638892

Equation B11 is then used to calculate S'_y.

$$S'_y = \sqrt{\frac{1}{n} + \frac{(X_c - \overline{X})^2}{\Sigma X_i^2 - \frac{(\Sigma Xi)^2}{N}}} \qquad \text{(Eq.B11)}$$

n = 40 (all Test Points used in the calculation)

$$\overline{X} = \frac{\Sigma(X_i)}{4} = \frac{0.00737628}{4} = 0.00184407 \qquad \text{(Eq.B12)}$$

when four test temperatures are used in the calculations.