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Road vehicles — Heavy commercial vehicles and buses – Mass moment of inertia measurement

Véhicules routiers — Véhicules utilitaires lourds et autobus — Mesure du moment d'inertie de masse

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Foreword

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The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular, the different approval criteria needed for the different types of ISO documents should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see www.iso.org/directives).

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For an explanation of the voluntary nature of standards, the meaning of ISO specific terms and expressions related to conformity assessment, as well as information about ISO's adherence to the World Trade Organization (WTO) principles in the Technical Barriers to Trade (TBT), see www.iso.org/iso/foreword.html.

This document was prepared by Technical Committee ISO/TC 22, *Road vehicles*, Subcommittee SC 33, *Vehicle dynamics and chassis components*.

Any feedback or questions on this document should be directed to the user's national standards body. A complete listing of these bodies can be found at www.iso.org/members.html.

Introduction

Methods are presented for determining the roll (I_{xx}), pitch (I_{yy}), and yaw (I_{zz}) mass moments of inertia (MOI) and roll-yaw (I_{xz}) product of inertia (POI) of an individual vehicle unit about the vehicle unit axis system and centre of gravity reference point. I_{xx}, I_{yy}, I_{zz}, and I_{xz} are fundamental mass properties that provide information on a vehicle's mass distribution and rotational acceleration responses to applied forces. The I_{xy} and I_{yz} components of the inertia tensor are less significant to vehicle dynamics and are not addressed in this document. The MOIs are determined using a pendulum device with measurements of oscillation period and reaction forces. The location of the vehicle unit's centre of gravity (CG) reference point is required beforehand. Knowledge of a vehicle unit's mass moments of inertia supports vehicle modelling work, design validation, and planning for other dynamic tests yet to be performed.

Performing measurements for MOI determination of heavy commercial vehicles and buses may be challenging in practice due to the wide variety of vehicles that vary significantly in terms of weight, size, and number of axles. Adaptability of a heavy vehicle MOI facility's layout is an important attribute.

Road vehicles — Heavy commercial vehicles and buses – Mass moment of inertia measurement

1 Scope

This document provides standard methods for determining a vehicle's roll, pitch, and yaw mass moments of inertia (MOI) and roll-yaw product of inertia (POI). It applies to heavy vehicles, that is commercial vehicles and buses as defined in ISO 3833 (trucks and trailers with maximum weight above 3,5 tons and buses and articulated buses with maximum weight above 5 tons, according to ECE and EC vehicle classification, categories M3, N2, N3, O3 and O4). Mass moment of inertia measurements are performed separately for each single unit.

2 Normative references

The following documents are referred to in the text in such a way that some or all of their content constitutes requirements of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 612, *Road vehicles — Dimensions of motor vehicles and towed vehicles — Terms and definitions*

ISO 8855, *Road vehicles — Vehicle dynamics and road-holding ability — Vocabulary*

ISO 15037-2, *Road vehicles — Vehicle dynamics test methods — Part 2: General conditions for heavy vehicles and buses*

3 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO 8855, ISO 15037-2 and the following apply.

ISO and IEC maintain terminology databases for use in standardization at the following addresses:

— ISO Online browsing platform: available at https://www.iso.org/obp

— IEC Electropedia: available at https://www.electropedia.org/

3.1
stable pendulum
pendulum apparatus for supporting a vehicle on a nominally planar surface where the combined vehicle and pendulum CG height is below the pivot point

3.2
unstable pendulum
pendulum apparatus for supporting a vehicle on a nominally planar surface where the combined vehicle and pendulum CG height is above the pivot point

3.3
torsional pendulum
pendulum apparatus where the restoring force is torsion

3.4
multi-filar torsional pendulum
torsional pendulum (3.3) with multiple vertical wires, cables or chains supporting the vehicle and test *platform* (3.5) where the torsional restoring force is due to gravity and spring force in twisting cables

3.5
platform
nominally planar surface of the pendulum on which the vehicle unit or trailer is parked

3.6
vehicle restraint
device used to restrain vehicle motion on the test *platform* (3.5)

3.7
tare MOI
MOI determined for the test fixture only, with no test vehicle

3.8
vehicle MOI
I_v
MOI determined for the vehicle only

3.9
platform MOI
I_p
MOI determined for the *platform* (3.5) only

3.10
roll MOI
I_{xx}
mass MOI about the vehicle X_V axis

3.11
pitch MOI
I_{yy}
mass MOI about the vehicle Y_V axis

3.12
yaw MOI
I_{zz}
mass MOI about the vehicle Z_V axis

3.13
roll-yaw POI
I_{xz}
mass product of inertia coupling between the X_V and Z_V axes

3.14
radius of gyration
k_r
distance from the axis of rotation to a point where the total mass of the body may be concentrated, so that the MOI about the axis remains the same

4 Principles

This document specifies methods to determine the vehicle's roll, pitch, and yaw MOIs and roll-yaw POI about the vehicle axis system originating from the vehicle CG reference point. Based on the vehicle axis system defined in ISO 8855, the MOI determinations described in this document include I_{xx}, I_{yy}, I_{zz}, and I_{xz}.

The I_{xx} and I_{yy} MOIs are determined using a stable or unstable pendulum fixture, where the period of oscillation of the fixture and fixture with the vehicle are measured to calculate the MOIs. The I_{zz} MOI is determined using a torsional or multi-filar torsional pendulum, where again the period of oscillation is measured. The I_{xz} POI is determined using a torsional pendulum with an integrated roll axis, where the roll axis is constrained and the roll reaction moment is measured.

The accuracy of MOI measurements is dependent on vehicle condition during measurement, measurement equipment accuracy, potential movement of heavy sprung or unsprung masses within the vehicle, such as engine and transmission assemblies and suspensions, and movement of the vehicle on the platform during the measurement process. Potential movement of fuel, coolants, and oils will also affect measurement accuracy.

5 Variables

5.1 Reference system

The reference system specified in ISO 15037-2 shall apply.

5.2 Variables to be measured

With the vehicle at the load condition specified for the test, measure and record the following in accordance with the dimensions given in ISO 612 and ISO 8855:

— W_v total vehicle load (or weight);

— W_p load of the platform including the restraint components;

— h pivot height from the platform surface;

— h_p platform plus restraints vertical CG distance below (or above) the pivot axis, including the roll pivot axis for I_{xz} measurement;

— h_v vehicle vertical CG distance below (or above) the pivot axis, including the roll pivot axis for I_{xz} measurement;

— h_t system (platform, restraints, and vehicle) vertical CG distance below (or above) the pivot axis;

— θ_p pitch angle of the platform relative to the gravity vector (positive for the front of the vehicle pitched down);

— $\theta_{p,\text{ static}}$ static pitch angle of the platform relative to the gravity vector (positive for the front of the platform pitched down);

— $\varphi_{p,\text{ static}}$ static roll angle of the platform relative to the gravity vector (positive for the platform rolled to the left);

— T_p platform (plus restraints) undamped period of oscillation;

— T_t total system (platform, restraints and vehicle) undamped period of oscillation;

— X longitudinal offset displacement of the vehicle CG relative to the platform centreline (positive for forward vehicle displacement);

— Y lateral offset displacement of the vehicle CG relative to the platform centreline;

— n number of oscillations of the platform;

— R radial distance on a multi-filar pendulum from the yaw axis centroid to the vertical support cables;

— L length of the multi-filar pendulum vertical support cables;

— ψ_p platform yaw angle amplitude;

— ψ_v vehicle yaw angle amplitude;

— L_u unstable pendulum moment arm length from reaction spring to pivot;

— K_u unstable pendulum reaction spring stiffness (force/displacement);

— K_p torsional pendulum reaction spring stiffness (moment/angle);

— M_x roll reaction moment for the torsional pendulum used for I_{xz} determination;

— M_{meas} measured roll reaction moment for the torsional pendulum used for I_{xz} determination.

6 Measuring equipment

The measuring equipment, transducer installation, data processing, and typical operating ranges shall be in accordance with ISO 15037-2.

Table 1 — Variables, typical operating ranges and recommended maximum errors of variables not listed in ISO 15037-2 for MOI measurement

Variable	Typical operating range	Recommended maximum errors of the combined transducer and recorder system
Suspension air-spring inflation pressure	(500–1 000) kPa	15 kPa
Vehicle, axle or track load	Up to 40 000 kg (392 400 N)	0,2 %
Distance	≤2 000 mm	±2 mm
	>2 000 mm	±0,05 %
Angles	±5°	±0,05°
Static distance X, Y	±20 mm	±3,0 mm
Dynamic Distance X(t), Y(t)	±10 mm	±0,5 mm
Time period	2 min	±1,0 ms
Roll moment	(0–1 500) N-m	0,5 %

7 Test conditions

7.1 General

The limits and specifications indicated below shall be maintained during the test. Any deviations shall be identified in the test report.

7.2 Ambient conditions

The surface shall be clean and dry. If the test is conducted outdoors, the ambient wind speed is recommended to be less than 1 m/s. Ambient temperature shall be recorded if the test is conducted outdoors.

7.3 Test surface

The test surface, when applicable, should be in accordance with ISO 15037-2 and the surface should be stiff enough to avoid surface deformation when measuring the vehicle.

7.4 Test vehicle

The load condition shall be reported as described in ISO 15037-2. Tyre pressures, suspension setting (if applicable) and load condition shall be recorded.

On vehicles with multiple adjustable seats or other devices such as beds, adjust the items to a mid-travel position (longitudinal and vertical) and adjust the seat back torso angle to the manufacturers designated specification or as close as possible to 15°. The positions shall be reported.

On vehicles with steering wheel reach and rake, the position shall be reported.

7.5 Operating and other liquids

The fuel tanks shall be completely full or empty, including the urea tanks. Fuel motion within an unfilled fuel tank can have an adverse effect on the results. If the displacement of other liquids carried on the vehicle (operating and otherwise), such as engine oil, is expected to influence the results, precautions should be taken to fill the fluid tanks, drain the fluids, or note the potential issue. Tank conditions (empty or full) and locations shall be reported.

Note any leaking fluids during vehicle inclination.

7.6 Loading conditions, suspension and mechanical parts

Vehicle payload shall be held in place to avoid displacement due to inclination or yawing of the vehicle.

If the vehicle has a suspended cab or semi-suspended cab, the cab shall be locked at its standard height when the vehicle is in a horizontal plane with no driver in the cab. Other components with flexible mounting may need to be constrained as well, if deflection will adversely influence the results.

Immediately prior to each test event, all self-regulating suspensions shall be adjusted such that they are at the proper ride height or, in the case of the suspensions for certain auxiliary axles, at the prescribed inflation pressure. Initial ride height of each suspension shall be reported.

Tyre condition and pressure shall be in accordance with vehicle manufacturer recommendations and ISO 15037-2. In case a range is specified for tyre pressure, the highest-pressure value should be selected to minimize tyre deflection.

Suspended components such as the engine, gearbox, and axles may move laterally and /or longitudinally when tilting or yawing the vehicle. Such displacements may influence measurement accuracy and should be noted accordingly.

8 Determination of the I_{xx}, I_{yy}, I_{zz}, and I_{xz} mass moments of inertia

8.1 General

Two methods are presented for measuring the parameters needed to calculate each of the I_{xx}, I_{yy}, and I_{zz} MOIs. The methods vary by the type of pendulum device used. A single method is presented for determining the I_{xz} POI. In general, the pendulum fixtures are mechanically re-configurable to enable measurement of various MOIs. Reconfiguration might include simply repositioning the vehicle on the platform to align the vehicle longitudinal or lateral axis with the pendulum rotational axis and adjusting pivot and spring hardware to facilitate different MOI and POI measurements.

8.1.1 Platform levelness

The platform's empty, static equilibrium position, $\theta_{p,\ static}$ and $\varphi_{p,\ static}$, should be checked via inclinometers to verify its levelness. It is recommended that the empty platform is level within 0,1° for best results and no more than 0,5°.

8.1.2 Platform weight and stiffness

It is recommended that the platform be as light as possible compared to the weight of the vehicle under test, while providing sufficient stiffness to avoid measurement errors due to deflection. When loaded with the test vehicle, it is recommended that the platform deflection is less than 2,5 mm at the vehicle's CG (as measured from the pivot when compared to the unloaded platform). Significant measurement errors are introduced when the deflection is greater than this, although corrections for deflection can be made. In general, a stiff platform is more important than a light platform.

8.1.3 Pivot location

The MOI platform's pivot height whether for a stable or unstable (inverted) compound pendulum is very important for measurement accuracy. For any given vehicle, an optimal pivot height exists that minimizes the MOI measurement error. Except for yaw pivots, the pivot height should be adjusted so that it is located at a height between the test vehicle's CG height and the vehicle's radius of gyration, k_r, about the axis in question (approximately half the wheelbase for typical trucks)[2], [3]. Measurement error is reduced by increasing the amplitude of the platform pitch angle (remaining less than 5°) while simultaneously reducing the period[2]. Trade-offs between pitch angle and period are achieved by adjusting the pivot height, and experimentation will likely be required to produce the desired results. For test fixtures intended to measure a wide range of vehicle sizes and weights, the pivot height should be adjustable.

For stable pendulum measurements, experience has indicated satisfactory pivot heights, h_v, for trucks on the order of 635 mm[2]. It is recommended that h_v is not smaller than 20 % of the h_p. If the pivot height is too small, period measurement error and vehicle acceleration motion relative to the platform are increased.

8.1.4 Vehicle weight

In practice, it is recommended that the test vehicle weighs at least 50 % of the carrier platform's weight. This limitation is due to the influence of the platform's weight on overall measurement error.

8.1.5 Vehicle/platform MOI comparison

The vehicle's anticipated moment of inertia values shall be within the capabilities of the facility. MOI accuracy is reduced when the vehicle's MOI is less than that of the platform (tare) MOI about the axis of interest. A comparison should be made after the vehicle MOI determination to verify.

8.1.6 Vehicle location on the platform

Position the vehicle on the platform with the vehicle longitudinal and lateral CG aligned with the platform centrelines. The platform shall be stabilized during the vehicle positioning process for safety and to avoid damage to the fixture and measurement equipment. For best results, it is recommended that the static offset distances X and Y are smaller than 13 mm. It is further recommended that the vehicle position, $X(t)$ and $Y(t)$, are measured dynamically when MOI measurements are conducted.

8.1.7 Vehicle restraints

Vehicle restraints shall be used to avoid vehicle pitch, roll and heave motion during testing relative to the platform. The restraints may be in the form of adjustable-height blocks or stands with straps constraining the vehicle against the blocks (or stands). Straps may also be used to restrain longitudinal, lateral, and yaw motion. Consideration of restraint selection should be given to make it easier to calculate the tare MOI with the restraints in their proper positions. Any restraint equipment utilised shall be weighed and the position on the platform recorded. Error contributions to the MOI determination about the pitch and roll axes for unrestrained heavy-class vehicles have been shown to be dominated by the motion of the vehicle in relation to the moving platform[4].

8.1.8 Platform oscillation amplitude

For best results, it is recommended that the maximum platform oscillation amplitude is in the range of 3° to 4°. Above this range, sin θ_p does not equal θ_p well enough for accurate measurement. The maximum angle should be less than four degrees, but large enough to produce sufficient cycles for proper period measurement. As a check of the measurements, tests may be performed at different amplitudes. If the results are affected by amplitude, the amplitude and/or apparatus are not working well. The use of a stationary indicator of the platform's initial amplitude (or a physical stop which can swing out of the way once platform motion has started) is recommended to achieve repeatable oscillation amplitudes.

8.1.9 Pivot bearing damping

Experience has shown that damping is approximately linear and proportional to velocity. The use of low friction pivot bearings, whether heavy-duty roller bearings for stable pendulums or high-capacity hydrostatic or air bearings for inverted or torsional pendulums, is recommended. Acceptable levels of damping may vary depending on the facility's desired measurement uncertainty. As a general rule, the damping error is insignificant if it takes more than 20 oscillations for the platform's swing amplitude to decay by a factor of 10.

The log decrement method may be used to estimate the damping ratio, as shown in Formulae (1) and (2).

$$d = \frac{1}{n} \ln\left(\frac{a_0}{a_n}\right) \tag{1}$$

$$\zeta = \left(1 + \left(\frac{2\pi}{d}\right)^2\right)^{-0,5} \tag{2}$$

where

- d is the log decrement (unitless);
- a_0 is the initial θ_p amplitude;
- a_n is the final θ_p amplitude after n periods;
- ζ is the damping ratio (unitless).

8.1.10 Oscillation period measurement

To excite the platform or platform/vehicle combination, manually apply an even force to the platform to start the excitation motion about the desired pivot axis without causing platform motion about the orthogonal axes. Allow a few oscillations to occur before recording data to allow any platform transient motion to steady out. Always perform a platform MOI tare. Do not assume past tares are applicable, as test condition variances occur, such as changes in fixturing/restraining devices, pivot height, and environmental conditions (temperature, air pressure). It is recommended to perform 10 test trials with 50 periods per test run, when possible, but no less than 10 periods. Allow the platform to settle and zero the electronic instrumentation between runs. This method allows observation of oscillation period variability. Also, averaging the period measurements improves the mean period's effective resolution.

The following methods may be used for determining the oscillation period.

1) Discrete timing: discrete oscillation period measurements are averaged for the test trials performed. Ensure that full periods are measured.

2) Continuous timing: when continuous time measurements of platform motion are recorded, the oscillation period can be determined analytically. Some common methods are described below.

   a) Damped-sine curve fit: the platform oscillation angle data are fit to a curve given by Formula (3) to determine the damped natural frequency. Of the methods described, this approach is perhaps the most accurate.

$$\theta_p(t) = \theta_{\text{p, static}} + a_0 \cdot e^{-\zeta \omega t} \cdot \sin(\omega t - \delta) \tag{3}$$

where

- $\theta_p(t)$ is the time-varying platform pitch angle (rad);

ω is the damped natural frequency (rad/s);

t is time (s);

δ is the phase angle (rad).

b) Fourier analysis: a Fast Fourier Transform (FFT) analysis of the results may be used to estimate the oscillation period. However, this approach may not provide sufficient frequency resolution to produce an accurate result due to limitations on sampling rates and FFT block size. The resolution of the FFT may be improved in the frequency range of interest via filtering and decimating the data. This method also produces good repeatability and is good for processing noisy data.

c) Zero-crossings: the period of oscillation can also be determined by observing zero-crossings of the angle or rate data. Data interpolation is used to establish the crossing times. This procedure may be problematic with noisy data signals or improperly filtered data.

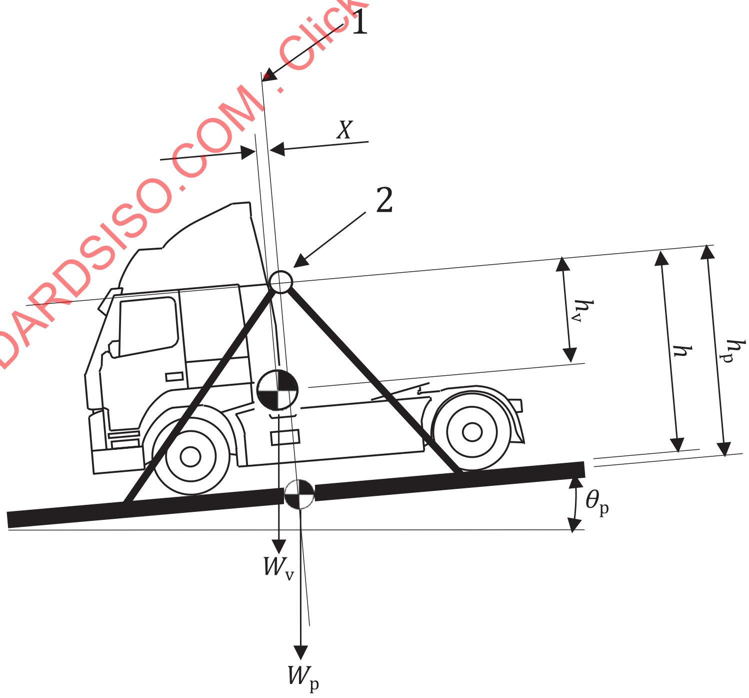
The undamped period of oscillation, $T_{undamped}$, is computed using Formula (4).

$$T_{undamped} = \frac{2\pi}{\omega\sqrt{1-\zeta^2}} \tag{4}$$

8.2 Determination of I_{xx} and I_{yy} using a stable pendulum

8.2.1 General guidance

Figure 1 illustrates a typical stable pendulum used to determine I_{xx} and I_{yy}. The fixture consists of a support structure (not shown in Figure 1), low friction bearings at the pivot axis, links from the pivot points to the vehicle platform, and a vehicle platform. Oscillation periods of the platform and platform plus test vehicle are measured to compute the I_{xx} and I_{yy} MOIs.



Key

1 centreline

2 pivot point

Figure 1 — Stable pendulum test fixture moving components for I_{yy} measurement orientation

8.2.2 Procedure

a) From preliminary testing results, record all pertinent physical dimensions and weights of the test vehicle and platform, including any restraints, and vehicle CG locations. ISO 19380 may be used as a guide.

b) Configure the stable pendulum for the intended MOI measurement (I_{xx} or I_{yy}).

c) Record the oscillation period, T_p, and dynamic pitch angle, $\theta_p(t)$, (or dynamic pitch rate if desired) of the empty platform (tare), with any necessary vehicle restraints in position, following the guidance provided in 8.1.

d) Position the test vehicle on the stable pendulum platform in the intended orientation to measure I_{xx} or I_{yy}, with the vehicle CG (longitudinal and lateral) aligned as closely as possible (with reasonable effort) with the platform centrelines (within 13 mm is recommended). Record the vehicle's initial X and Y position from the longitudinal and lateral axes of the fixture.

e) Restrain the test vehicle's sprung mass to the test platform to minimize vehicle dynamic movement during testing, as described in 8.1.7.

f) Record the oscillation period of the platform and vehicle system (T_t) based on platform measurements, as well as the platform dynamic pitch angle, $\theta_p(t)$, (or dynamic pitch rate, $\dot{\theta}_p$, if desired) with vehicle restraints in position as described in 8.1. Simultaneously measure the dynamic position of the vehicle relative to the platform, $X(t)$ or $Y(t)$ as appropriate, for the MOI measurement of interest.

g) Repeat the procedure for measurements associated with the other MOI (I_{xx} or I_{yy}).

8.2.3 Determination of I_{xx} and I_{yy}

Formulae (5) and (6) are used to compute the vehicle's roll (I_{xx}) and pitch (I_{yy}) MOIs, based on the orientation of the vehicle on the platform. Formula (6) includes corrections for the vehicle's relative displacement motion[5], [6].

$$I_p = \frac{T_p^2}{4\pi^2} W_p h_p \tag{5}$$

$$I_v = \frac{T_t^2}{4\pi^2}\left[\left(W_p + W_v\right)h_t - R_p W_v\right] + R_p \frac{W_v}{g} h_v - I_p - \frac{W_v}{g} h_v^2 \tag{6}$$

where

I_p is the platform inertia about the pivot (not about the platform CG) including the vehicle restraints. The restraint inertias may be accounted for in two ways. I_p may be measured directly from Formula (5) with all restraints in their test positions, or I_p may be derived from an empty platform measurement with no restraints and with the inertias of the restraints (about the pivot) added analytically. If the restraints are more than 10 % of the platform weight or if the restraints have significant inertias about their own CGs, the first method is preferred.

R_p is the ratio of vehicle longitudinal motion on the platform, $X(t)$ *or* $Y(t)$, to the platform angle, $\theta_p(t)$ (distance unit/rad) during the measurement interval.

If corrections for the vehicle relative motion are not practical and R_p is believed to be very small, Formula (6) can be simplified accordingly by setting R_p to zero.

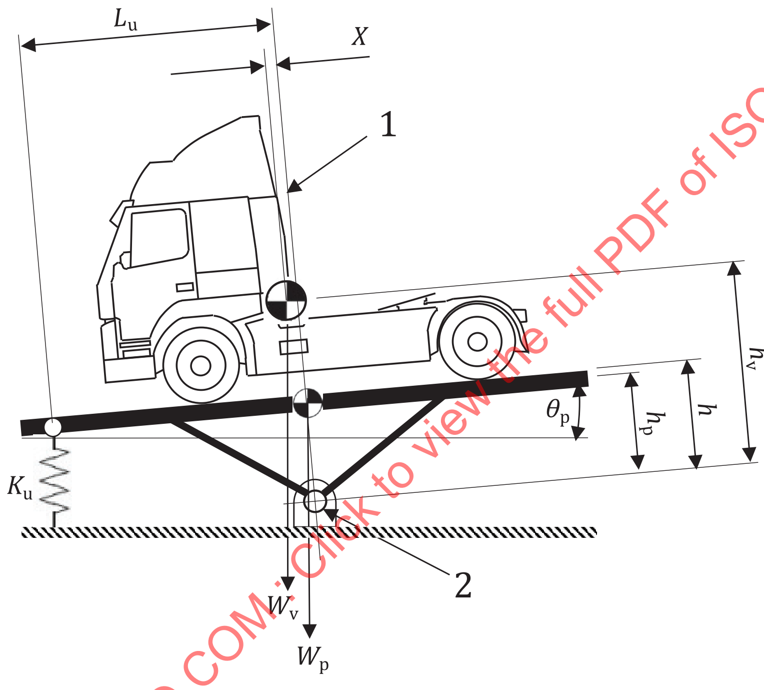
8.2.4 Data presentation

It is recommended that test results are presented in tabular form to include as a minimum the vehicle identification, configuration, mass/weight and CG position, platform identification, pivot heights h_p, h_v, and h_t, damping ratio, undamped period, and number of trials and periods measured.

8.3 Determination of I_{xx} and I_{yy} using an unstable pendulum

8.3.1 General guidance

Conceptually, this type of pendulum consists of a platform with the pivot axis below the platform and with a linear spring providing the restoring (stabilizing) force, as shown in Figure 2. It is recommended that the reaction springs' rate is selected to achieve a platform/vehicle system oscillation frequency of approximately 0,5 Hz[3], based on the vehicle orientation.



Key

1 centreline

2 pivot point

Figure 2 — Schematic of unstable pendulum test fixture for I_{yy} measurement orientation

8.3.2 Procedure

a) From preliminary testing results, record all pertinent physical dimensions and weights of the test vehicle and platform, including any restraints, and vehicle CG locations. ISO 19380 may be used as a guide.

b) Configure the unstable pendulum for the intended MOI measurement (I_{xx} or I_{yy}).

c) Record the oscillation period, T_p, and dynamic pitch angle, $\theta_p(t)$, (or dynamic pitch rate if desired) of the empty platform (tare), with any necessary vehicle restraints in position, following the guidance provided in 8.1.

d) Position the test vehicle on the unstable pendulum platform in the intended orientation to measure I_{xx} or I_{yy}. with the vehicle CG (longitudinal and lateral) aligned as closely as possible (with reasonable effort) with the platform centrelines. Record the vehicle's initial *X* and *Y* position from the longitudinal and lateral axes of the fixture.

e) Restrain the test vehicle's sprung mass to the test platform to minimize vehicle dynamic movement during testing, as described in 8.1.7.

f) Record the oscillation period of the platform and vehicle system (T_t) based on platform measurements, as well as the platform dynamic pitch angle, $\theta_p(t)$, (or dynamic pitch rate, $\dot{\theta}_p$, if desired) with vehicle restraints in position as described in 8.1. Simultaneously measure the dynamic position of the vehicle relative to the platform, $X(t)$ or $Y(t)$, as appropriate, for the MOI measurement of interest.

g) Repeat the procedure for measurements associated with the other MOI (I_{xx} or I_{yy}).

8.3.3 Calculation of I_{xx} and I_{yy}

Formulae (7) and (8) are used to compute the vehicle's roll (I_{xx}) and pitch (I_{yy}) inertias and contain corrections for the vehicle's relative displacement motion[6]. If corrections for the vehicle relative motion are not practical and R_p is believed to be very small, Formula (8) can be simplified accordingly by setting R_p to zero.

$$I_p = \frac{T_p^2}{4\pi^2}\left[K_u L_u^2 - W_p h_p\right] \tag{7}$$

$$I_v = \frac{T_t^2}{4\pi^2}\left[K_u L_u^2 - \left(W_p h_p + W_v h_v\right) - R_p W_v\right] + R_p \frac{W_v}{g} h_v - I_p - \frac{W_v}{g} h_v^2 \tag{8}$$

where I_p is the platform inertia about the pivot (not about the platform CG) including the vehicle restraints. The inertia of the restraints can be calculated or accounted for as with the stable pendulum described previously.

In Formulae (7) and (8), the $K_u L_u{}^2$ term represents the rotational spring stiffness about the pivot. The units for this term are N-m/radian. A single extension or compression spring can be used at distance L_u, or a set of springs can be used at the same or different L_u values. In all cases the force of the spring must act on a line perpendicular to the lever arm about the pivot. Alternatively, one or more torsional springs can be used. The total spring stiffness is the most important factor. The springs must be linear (i.e. torque is proportional to the angular displacement) in both rotational directions about the platform zero point.

The rotational spring stiffness can be determined in two ways. First, the stiffness can be measured directly by applying known torques (from actuators or weights) to the platform and measuring the platform rotational displacement. Alternatively, the stiffness can be determined by measuring the period of the empty platform and then measuring the period of the platform with a rigid object of known inertia (about its own CG), weight, and CG location on the platform (fixedly attached to the platform). It is recommended that the period of oscillation with the object is a least 50 % greater than the period with no object and that the tests are repeated with at least two different objects to verify results. Formula (9) is then used to compute the rotational spring stiffness ($K_u L_u{}^2$) for the second method.

$$K_u L_u{}^2 = \frac{1}{\left(T_t{}^2 - T_p{}^2\right)}\left[4\pi^2\left(I_b + \frac{W_b}{g} h_b^2\right) + W_b h_b T_t^2\right] + W_p h_p \tag{9}$$

where

b subscripts relate to the object;

p subscripts relate to the platform;

t subscripts relate to the system of the platform and object.

8.3.4 Data presentation

It is recommended that test results are presented in tabular form to include as a minimum the vehicle identification, configuration, mass/weight and CG position, platform identification, pendulum spring rates, pivot heights h_p, h_v, and h_t, damping ratio, undamped period, and number of trials and periods measured.

8.4 Determination of I_{zz} using a torsional pendulum

8.4.1 General guidance

This type of pendulum consists of a platform with the pivot axis oriented vertically and centred with the platform dimensions. A linear torsional spring system, generally beneath the platform, provides the restoring (stabilizing) force. It is recommended that the reaction springs' rate is selected to achieve a platform/vehicle system oscillation frequency of approximately 0,17 (1/6) to 0,5 Hz[4].

8.4.2 Procedure

a) From preliminary testing results, record all pertinent physical dimensions and weights of the test vehicle and platform, including any restraints, and vehicle CG locations. ISO 19380 may be used as a guide.

b) Configure the torsional pendulum and instrumentation for the I_{zz} measurement.

c) Record the oscillation period and dynamic yaw angle, $\psi_p(t)$, (or dynamic yaw rate if desired) of the empty platform (tare), with any necessary vehicle restraints in position, following the general guidance provided in 8.1.

d) Position the test vehicle on the torsional pendulum platform with the vehicle CG (longitudinal and lateral) aligned as closely as possible (with reasonable effort) with the platform centrelines. Record the vehicle's initial *X* and *Y* position from the longitudinal and lateral axes of the fixture.

e) Restrain the test vehicle's sprung mass to the test platform to minimize vehicle dynamic movement during testing, as described in 8.1.7.

f) Record the oscillation period of the platform and vehicle system (T_t) based on platform measurements, as well as the platform dynamic yaw angle, $\psi_p(t)$, (or dynamic yaw rate if desired) with vehicle restraints in position as described in 8.1. Simultaneously measure the dynamic yaw angle of the vehicle, $\psi_v(t)$, or the relative angle between the platform and vehicle, ($\psi_p(t)$- $\psi_v(t)$).

8.4.3 Calculation of I_{zz}

Formulae (10) and (11) are used to compute the vehicle's yaw (I_{zz}) inertia and contain a correction for the vehicle's relative motion[5].

$$I_p = \frac{T_p^2 K_p}{4\pi^2} \tag{10}$$

$$I_v = \frac{\psi_p \left[\frac{T_t^2 K_p}{4\pi^2} - I_p \right]}{\left(\psi_p + \left(\psi_p - \psi_v \right) \right)} \tag{11}$$

where I_p is the platform inertia about the pivot including the vehicle restraints. The inertia of the restraints can be calculated or accounted for as with the stable pendulum described previously.

In Formulae (10) and (11), the K_p term represents the total rotational spring stiffness about the pivot. The units for this term are N-m/radian. The springs used must be linear (i.e. torque is proportional to the angular displacement) in both rotational directions about the platform zero point.

The rotational spring stiffness can be determined in two ways. First, the stiffness can be measured directly by applying known torques to the platform and measuring the platform rotational displacement. Alternatively, the stiffness can be determined by measuring the period of the empty platform and then measuring the period of the platform with a rigid object of known inertia (about the pivot and fixedly attached to the platform). It is recommended that the period of oscillation with the object is a least 50 % greater than the period with no object and that the tests are repeated with at least two different objects to verify results. Formula (12) is then used to compute the rotational spring stiffness K_p for the second method.

$$K_\mathrm{p} = \frac{4\pi^2 I_\mathrm{b}}{\left(T_\mathrm{t}^2 - T_\mathrm{p}^2\right)} \qquad (12)$$

where the b subscript relates to the object, the p subscript relates to the platform, and the t subscript relates to the system of the platform and object.

8.4.4 Data presentation

It is recommended that test results are presented in tabular form to include as a minimum the vehicle identification, configuration, mass/weight and CG position, platform identification and inertia (with restraints), torsional spring stiffness, damping ratio, undamped period, and number of trials and periods measured.

8.5 Determination of I_{xz} using a torsional pendulum

8.5.1 General guidance

Measure the product of inertia I_{xz} using a torsional pendulum fixture (described in 8.4) with an integrated, constrained roll axis (referring to the fixture). The roll constraint shall be stiff and includes a load cell(s) to measure the roll couple induced during yaw motion. This test is generally performed simultaneously with the yaw MOI measurement (8.4).

8.5.2 Procedure

The same general procedures as 8.4 are followed, except that the dynamic roll moment, M_{meas} (t), is also measured during the test.

8.5.3 Calculation of I_{xz}

Assuming that the platform and restraints POI are zero, Formulae (13) and (14) are used to compute the vehicle's roll-yaw (I_{xz}) product of inertia[4], [5].

$$I_\mathrm{xz} = -\frac{M_\mathrm{x}}{\ddot{\psi}_\mathrm{p}} \qquad (13)$$

$$M_\mathrm{x} = M_\mathrm{meas,\,max} - \frac{\left(W_\mathrm{p} h_\mathrm{p} X_\mathrm{p} + W_\mathrm{v} h_\mathrm{v} X_\mathrm{v}\right)}{g}\ddot{\psi}_\mathrm{p} \qquad (14)$$

where

$\ddot{\psi}_\mathrm{p}$ is the platform yaw angular acceleration at maximum displacement (rad/s^2);

$M_\mathrm{meas,\,max}$ is the measured reaction moment at maximum displacement;