INTERNATIONAL STANDARD

ISO 16063-43

First edition
2015-11-15

Corrected version
2016-07-15

Methods for the calibration of vibration and shock transducers —

Part 43:
Calibration of accelerometers by model-based parameter identification

Méthodes pour l'étalonnage des transducteurs de vibrations et de chocs —

Partie 43: Étalonnage des accéléromètres par identification des paramètres à base de modèle

Reference number
ISO 16063-43:2015(E)

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Foreword

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The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular the different approval criteria needed for the different types of ISO documents should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see www.iso.org/directives).

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For an explanation on the meaning of ISO specific terms and expressions related to conformity assessment, as well as information about ISO's adherence to the World Trade Organization (WTO) principles in the Technical Barriers to Trade (TBT) see the following URL: www.iso.org/iso/foreword.html.

The committee responsible for this document is ISO/TC 108, *Mechanical vibration, shock and condition monitoring*, Subcommittee SC 3, *Use and calibration of vibration and shock measuring instruments*.

This corrected version of ISO 16063-43:2015 incorporates the following corrections:

— Formulae (26) and (32) corrected;

— symbol i used for the imaginary unit; symbol χ used where necessary; symbols R and J used to indicate real and imaginary parts;

— Figures 3 and 4 brought in line with the formulae in the text;

— editorial improvements, including "transducer" used instead of "sensor" or "pick-up";

— Reference [6] corrected.

— application of the ISO/IEC Directives, Part 2, 2016.

A list of all the parts in the ISO 16063 series can be found on the ISO website.

Introduction

The ISO 16063 series describes in several of its parts (ISO 16063-1, ISO 16063-11, ISO 16063-13, ISO 16063-21 and ISO 16063-22) the devices and procedures to be used for calibration of vibration transducers. The approaches taken can be divided in two classes: one for the use of stationary signals, namely sinusoidal or multi-sinus excitation; and the other for transient signals, namely shock excitation. While the first provides the lowest uncertainties due to intrinsic and periodic repeatability, the second aims at the high intensity range where periodic excitation is usually not feasible due to power constraints of the calibration systems.

The results of the first class are given in terms of a complex transfer sensitivity in the frequency domain and are, therefore, not directly applicable to transient time domain application.

The results of the second class are given as a single value, the peak ratio, in the time domain that neglects (knowingly) the frequency-dependent dynamic response of the transducer to transient input signals with spectral components in the resonance area of the transducer's response. As a consequence of this "peak ratio characterization", the calibration result might exhibit a strong dependence on the shape of the transient input signal applied for the calibration and, therefore, from the calibration device.

This has two serious consequences:

a) The calibration with shock excitation in accordance with ISO 16063-13 or ISO 16063-22 is of limited use as far as the dissemination of units is concerned. That is, the shock sensitivities S_{sh} determined by calibrations on a device in a primary laboratory might not be applicable to the customer's device in the secondary calibration lab, simply due to a different signal shape and thus spectral constitution of the secondary device's shock excitation signal.

b) A comparison of calibration results from different calibration facilities with respect to consistency of the estimated measurement uncertainties, e.g. for validation purposes in an accreditation process, is not feasible if the facilities apply input signals of differing spectral composition.

The approach taken in this document is a mathematical model description of the accelerometer as a dynamic system with mechanical input and electrical output, where the latter is assumed to be proportional to an intrinsic mechanical quantity (e.g. deformation). The estimates of the parameters of that model and the associated uncertainties are then determined on the basis of calibration data achieved with established methods (ISO 16063-11, ISO 16063-13, ISO 16063-21 and ISO 16063-22). The complete model with quantified parameters and their respective uncertainties can subsequently be used to either calculate the time domain response of the transducer to arbitrary transient signals (including time-dependent uncertainties) or as a starting point for a process to estimate the unknown transient input of the transducer from its measured time-dependent output signal (ISO 16063-11 or ISO 16063-13).

As a side effect, the method also usually provides an estimate of a continued frequency domain transfer sensitivity of the model.

In short, this document prescribes methods and procedures that enable the user to

— calibrate vibration transducers for precise measurements of transient input,

— perform comparison measurements for validation using transient excitation,

— predict transient input signals and the time-dependent measurement uncertainty, and

— compensate the effects of the frequency-dependent response of vibration transducers (in real time) and thus expand the applicable bandwidth of the transducer.

Methods for the calibration of vibration and shock transducers —

Part 43: Calibration of accelerometers by model-based parameter identification

1 Scope

This document prescribes terms and methods on the estimation of parameters used in mathematical models describing the input/output characteristics of vibration transducers, together with the respective parameter uncertainties. The described methods estimate the parameters on the basis of calibration data collected with standard calibration procedures in accordance with ISO 16063-11, ISO 16063-13, ISO 16063-21 and ISO 16063-22. The specification is provided as an extension of the existing procedures and definitions in those International Standards. The uncertainty estimation described conforms to the methods established by ISO/IEC Guide 98-3 and ISO/IEC Guide 98-3/Supplement 1.

The new characterization described in this document is intended to improve the quality of calibrations and measurement applications with broadband/transient input, like shock. It provides the means of a characterization of the vibration transducer's response to a transient input and, therefore, provides a basis for the accurate measurement of transient vibrational signals with the prediction of an input from an acquired output signal. The calibration data for accelerometers used in the aforementioned field of applications should additionally be evaluated and documented in accordance with the methods described below, in order to provide measurement capabilities and uncertainties beyond the limits drawn by the single value characterization given by ISO 16063-13 and ISO 16063-22.

2 Normative references

The following documents are referred to in the text in such a way that some or all of their content constitutes requirements of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 16063-11, *Methods for the calibration of vibration and shock transducers — Part 11: Primary vibration calibration by laser interferometry*

ISO 16063-13, *Methods for the calibration of vibration and shock transducers — Part 13: Primary shock calibration using laser interferometry*

ISO 16063-21, *Methods for the calibration of vibration and shock transducers — Part 21: Vibration calibration by comparison to a reference transducer*

ISO 16063-22, *Methods for the calibration of vibration and shock transducers — Part 22: Shock calibration by comparison to a reference transducer*

ISO/IEC Guide 98-3, *Uncertainty of measurement — Part 3: Guide to the expression of uncertainty in measurement (GUM:1995)*

ISO/IEC Guide 98-3/Supplement 1, *Uncertainty of measurement — Part 3: Guide to the expression of uncertainty in measurement (GUM:1995) — Supplement 1: Propagation of distributions using a Monte Carlo method*

3 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO 2041 apply.

ISO and IEC maintain terminological databases for use in standardization at the following addresses:

— IEC Electropedia: available at http://www.electropedia.org/

— ISO Online browsing platform: available at http://www.iso.org/obp

4 List of symbols

The symbols used in the formulae are listed in order of occurrence in the text.

$x, \dot{x}, \ddot{x}$	Output quantity of the respective transducer and its single and double derivative over time
δ	Damping coefficient of the model equation in the time domain
ω_0	Circular resonance frequency of the model
ρ	Electromechanical conversion factor
i	Imaginary unit, $\mathrm{i} = \sqrt{-1}$
H	Complex valued transfer function
S	Magnitude of the transfer function
ϕ	Phase of the transfer function
G	Reciprocal of the complex valued transfer function
μ	Parameter vector
S_m	Magnitude of the transfer function for a circular frequency, ω_m
ϕ_m	Phase of the transfer function for a circular frequency, ω_m
R	Real part of the complex valued transfer function
J	Imaginary part of the complex valued transfer function
y	Vector of real and imaginary parts of the measured transfer function
V_{y}	Covariance matrix of y
D	Coefficients matrix
$\hat{\mu}$	Vector of parameter estimates
$V_{\hat{\mu}}$	Covariance matrix of $\hat{\mu}$
S_0	Magnitude of the transfer function at low frequencies
A_{μ}	Transformation matrix for analytical uncertainty propagation
$V_{\rho,\omega_0,\delta}$	Covariance matrix of the model parameters
s	Frequency analogue in the s-domain (s-transform)

A	Acceleration in the s-domain
X	Output quantity of the respective transducer in the s-domain
z^{-1}	Back shift operator used in the bilinear transform (z-transform)
T	Sampling interval
a_k	Measured input acceleration sample at the time step k
x_k	Measured accelerometer output sample at the time step k
b, c_1, c_2, Λ	Model parameters in the case of discretized time domain data
v	Substitutional parameters for the time domain parameter estimation
$\hat{v}$	Estimates of v by weighted least squares fitting
$V_{\hat{v}}$	Covariance matrix of the estimated parameters $\hat{v}$
Ω	Circular frequency normalized to the sample rate
χ^2	Sum of weighted squared residuals
y_k	Calculated transducer output for the time step k based on estimated parameters
$\hat{b}, \hat{c}$	Best estimates of b and c (see 9.2)

5 Consideration of typical frequency response and transient excitation

A typical acceleration transducer has a complex frequency response. This is usually given in terms of magnitude and phase with a shape, as shown in Figure 1. The magnitude is given in arbitrary units (a.u.).

This response function is subsequently sampled with lowest uncertainties by a calibration method in accordance with ISO 16063-11 or ISO 16063-21 making use of periodic excitation.

In applications with transient input signals, the transducer is then exposed to broadband excitation in terms of the frequency domain. The response in this case cannot be calculated with the help of a single (complex) value like the transfer sensitivity. Rather, the response can be considered to be a sensitivity that is weighted by those components in the frequency response that are excited by the spectral contents of the input signal.

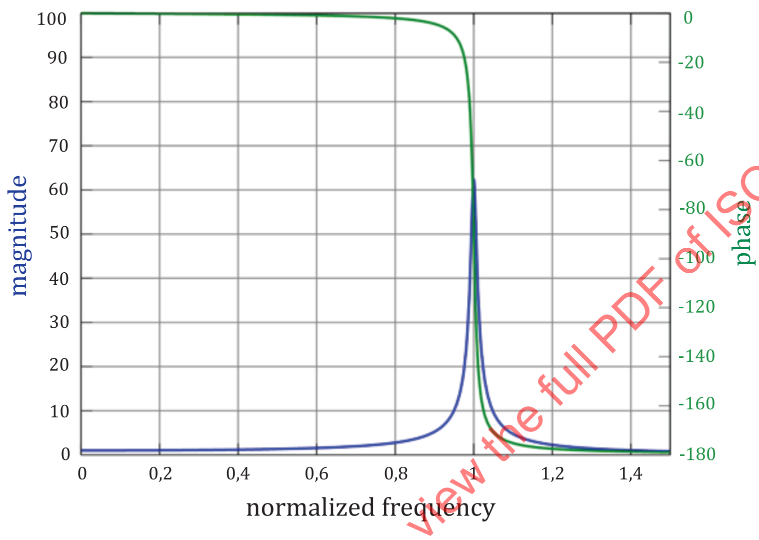
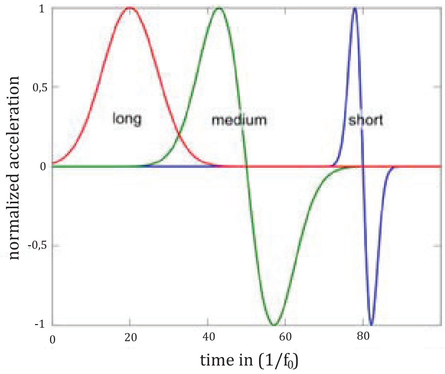
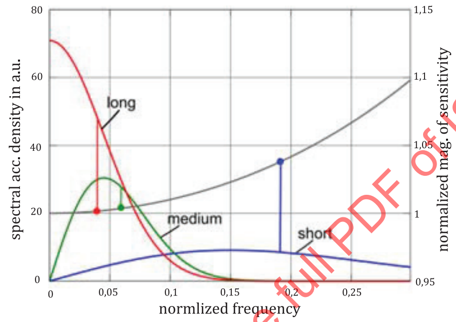


Figure 1 — Complex frequency response of a typical accelerometer in terms of magnitude of sensitivity (blue) and phase delay (green) over the normalized frequency

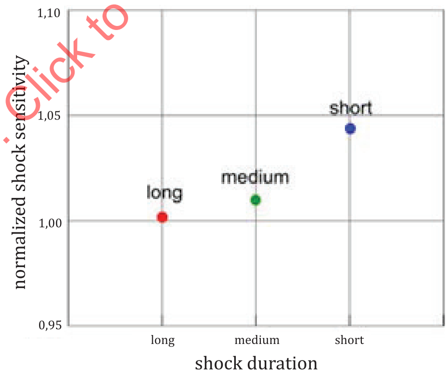
Figure 2 gives a pictorial representation of three examples of possible shock excitation signals and their respective spectra as compared to the frequency response of a typical transducer. It shows the projection of the centre of mass of the magnitude of the spectral density curve onto the sensitivity curve of a typical accelerometer. This demonstrates that a single value characterization of a transducer by shock calibration cannot sufficiently describe the dynamic behaviour.



a) Time domain representation of a long monopole (red), medium dipole (green), and short dipole (blue) shock



b) Frequency domain representation (magnitude) with the projection of the spectral centre point onto the sensitivity curve of a typical accelerometer response



c) Corresponding shock sensitivity (peak ratio) of a typical accelerometer

Figure 2 — Comparison of the characteristics of three different shock signals

6 General approach

The general idea behind "model-based parameter identification" is to describe the input/output behaviour of a transducer type of certain design and construction with the help of a dynamic mathematical model. The detailed properties of an individual transducer are represented in that model by a set of parameters. Associated with the set of estimates of the model parameters is a respective set of uncertainties. The aim of the calibration is to provide measurement results that allow for the mathematical estimation of this parameter set and the evaluation of corresponding uncertainties.

NOTE 1 The parameter sets can include functions of variables to cover temperature sensitivity or mass loading effects.

This general approach is not new, and is already well-established in the fields of science and engineering under the term "identification of dynamic systems". However, in the field of transducer calibration, special emphasis has to be put on the validation of the applicability of the methods used and on the reliable calculation of uncertainties and respective coverage intervals.

NOTE 2 In this document, the procedure of model-based parameter identification and further considerations is presented for a linear mass-spring-damper model of a seismic transducer. However, this is only one example. The same approach can be used for more complicated mathematical models as long as they can be described as linear time-invariant (LTI) systems.

7 Linear mass-spring-damper model

7.1 Model

According to the investigation described in References [1], [2] and [3], some accelerometers can be described by a simple linear mass-spring-damper model in their specified working range. That means they follow the general equation of motion of the form[2] as given in Formula (1):

$$\ddot{x} + 2\delta\omega_0\dot{x} + \omega_0^2 x = \rho\, a\left(t\right) \tag{1}$$

where

δ is the damping coefficient;

ω_0 is the circular resonance frequency of the system;

ρ is the electromechanical conversion factor.

This model describes the dynamic output $x(t)$ (e.g. charge or voltage) as a function of the acceleration input $a(t)$.

For such a linear system the transfer function $H(\mathrm{i}\omega)$ in the frequency domain is independent of the acceleration amplitude and is given in Formula (2):

$$H\left(\mathrm{i}\omega\right) = \frac{\rho}{\omega_0^2 + 2\mathrm{i}\omega\delta\omega_0 + \left(\mathrm{i}\omega\right)^2} = S\left(\omega\right)\cdot \mathrm{e}^{\mathrm{i}\phi\left(\omega\right)} \tag{2}$$

The inverse of this transfer function is given in Formula (3):

$$G(\mathrm{i}\omega) = H^{-1}(\mathrm{i}\omega) = \rho^{-1}\left(\omega_0^2 + 2\mathrm{i}\omega\delta\omega_0 - \omega^2\right) = S^{-1}(\omega)\cdot \mathrm{e}^{-\mathrm{i}\phi(\omega)} \tag{3}$$

where

$S(\omega)$ is the magnitude;

$\phi(\omega)$ is the phase of the response.

7.2 Identification by sinusoidal calibration data

7.2.1 Parameter identification

Starting from calibration measurements with sinusoidal excitation in accordance with, for example, ISO 16063-11 or ISO 16063-21, the frequency response $H(\mathrm{i}\omega)$ can be directly determined as described by Formula (2) taking into account the well-known frequency response of any conditioning amplifier.

NOTE The model assumes that any additional response function of a measuring amplifier is eliminated prior to the identification process, which is usually the case.

Substituting a parameter vector, as given in Formula (4):

$$\mu^{\mathrm{T}} = \left(\mu_1, \mu_2, \mu_3\right) = \left(\frac{\omega_0^2}{\rho}, \frac{2\delta\omega_0}{\rho}, \frac{1}{\rho}\right) \tag{4}$$

Formula (3) transforms into Formula (5):

$$G(\mathrm{i}\omega) = \frac{1}{H(\mathrm{i}\omega)} = \mu_1 + \mathrm{i}\omega\mu_2 - \omega^2\mu_3 = g^{\mathrm{T}}(\omega)\cdot\mu \tag{5}$$

where

$$g^{\mathrm{T}}(\omega) = \left(1, \mathrm{i}\omega, -\omega^2\right)$$

According to this relation, the parameter vector μ can be estimated by weighted linear least squares, where the weights are chosen according to the uncertainties known from the calibration procedures in accordance with ISO 16063-11 or ISO 16063-21 as follows.

Let $S_m = S(\omega_m)$, $\phi_m = \phi(\omega_m)$ denote the magnitude and the phase of the frequency response from calibration measurements with associated standard uncertainties $u(S_m)$, $u(\phi_m)$ at the frequencies ω_m, $m = 1, 2,..., L$. Then the real part $R\left(S^{-1}\cdot \mathrm{e}^{-\mathrm{i}\phi}\right)$ and imaginary part $J\left(S^{-1}\cdot \mathrm{e}^{-\mathrm{i}\phi}\right)$ are given by Formula (6):

$$R(S,\phi) = R\left(S^{-1}\cdot \mathrm{e}^{-\mathrm{i}\phi}\right) = S^{-1}\cos(\phi),\ J(S,\phi) = J\left(S^{-1}\cdot \mathrm{e}^{-\mathrm{i}\phi}\right) = -S^{-1}\sin(\phi) \tag{6}$$

This is, in principle, a nonlinear transform which should be adequately handled for uncertainty calculations by, for example, Monte Carlo methods (see ISO/IEC Guide 98-3/Supplement 1 for details).

However, given that the uncertainties of measurement are small enough, the direct propagation of uncertainties can be calculated in accordance with ISO/IEC Guide 98-3, as shown in Formula (7):

$$\begin{aligned} u^2(R_m) &= \frac{u^2(S_m)}{S_m^4}\cos^2(\phi_m) + \frac{u^2(\phi_m)}{S_m^2}\sin^2(\phi_m) \\ u^2(J_m) &= \frac{u^2(S_m)}{S_m^4}\sin^2(\phi_m) + \frac{u^2(\phi_m)}{S_m^2}\cos^2(\phi_m) \\ u(R_m,J_m) &= \frac{-u^2(S_m)}{S_m^4}\sin(\phi_m)\cos(\phi_m) + \frac{u^2(\phi_m)}{S_m^2}\sin(\phi_m)\cos(\phi_m) \end{aligned} \tag{7}$$

where $R_m = R(S_m, \phi_m)$ and $J_m = J(S_m, \phi_m)$

Then let Formula (8) be the transformed vector of the measurands:

$$y^{\mathrm{T}} = \left[R(S_1,\phi_1),\ldots,R(S_L,\phi_L),J(S_1,\phi_1),\ldots,J(S_L,\phi_L) \right] \tag{8}$$

With the assumption that S and ϕ are uncorrelated measurands, the $2L \times 2L$ covariance matrix V_{y} becomes Formula (9):

$$V_{\mathrm{y}} = \begin{pmatrix} u^2(R_1) & & 0 & u(R_1,J_1) & & 0 \\ & \ddots & & & \ddots & \\ 0 & & u^2(R_L) & & & u(R_L,J_L) \\ u(R_1,J_1) & & & u^2(J_1) & & 0 \\ & \ddots & & & \ddots & \\ 0 & & u(R_L,J_L) & 0 & & u^2(J_L) \end{pmatrix} \tag{9}$$

D is the $2L \times 3$ matrix of the real and imaginary parts of $g^{\mathrm{T}}(\omega)$, as given in Formula (10):

$$D = \begin{pmatrix} 1 & 0 & -\omega_1^2 \\ 1 & 0 & -\omega_2^2 \\ & \vdots & \\ 1 & 0 & -\omega_{\mathrm{L}}^2 \\ 0 & \omega_1 & 0 \\ 0 & \omega_2 & 0 \\ & \vdots & \\ 0 & \omega_{\mathrm{L}} & 0 \end{pmatrix} \tag{10}$$

The weighted least square estimate of the parameters can be calculated according to Formula (11):

$$\hat{\mu} = (D^{\mathrm{T}} V_{\mathrm{y}}^{-1} D)^{-1} D^{\mathrm{T}} V_{\mathrm{y}}^{-1} y \tag{11}$$

The uncertainties associated with the estimated parameter set $\hat{\mu}$, i.e. the covariance matrix, are given by Formula (12):

$$V_{\hat{\mu}} = \left(D^{\mathrm{T}} V_{\mathrm{y}}^{-1} D\right)^{-1} \tag{12}$$

The original model parameters can subsequently be calculated by transforming Formula (4) as Formula (13):

$$\begin{aligned} \rho &= \mu_3^{-1} \\ \omega_0 &= \sqrt{\frac{\mu_1}{\mu_3}} \\ \delta &= \frac{\mu_2}{\sqrt{\mu_1 \cdot \mu_3}} \end{aligned} \tag{13}$$

Sometimes it is more convenient to write Formula (2) in terms of $S_0 = \rho / \omega_0^2$ instead of ρ where S_0 describes the sensitivity for low frequencies. The corresponding parameter equation is given by Formula (14):

$$S_0 = \frac{1}{\mu_1} \tag{14}$$

Since the inverse transform from Formula (4) to Formula (13) is nonlinear the uncertainties associated with the model parameter set should be adequately handled for uncertainty calculations by, for example, Monte Carlo methods as described in Reference [1].

Figure 3 gives a flowchart representation of the whole analysis process.

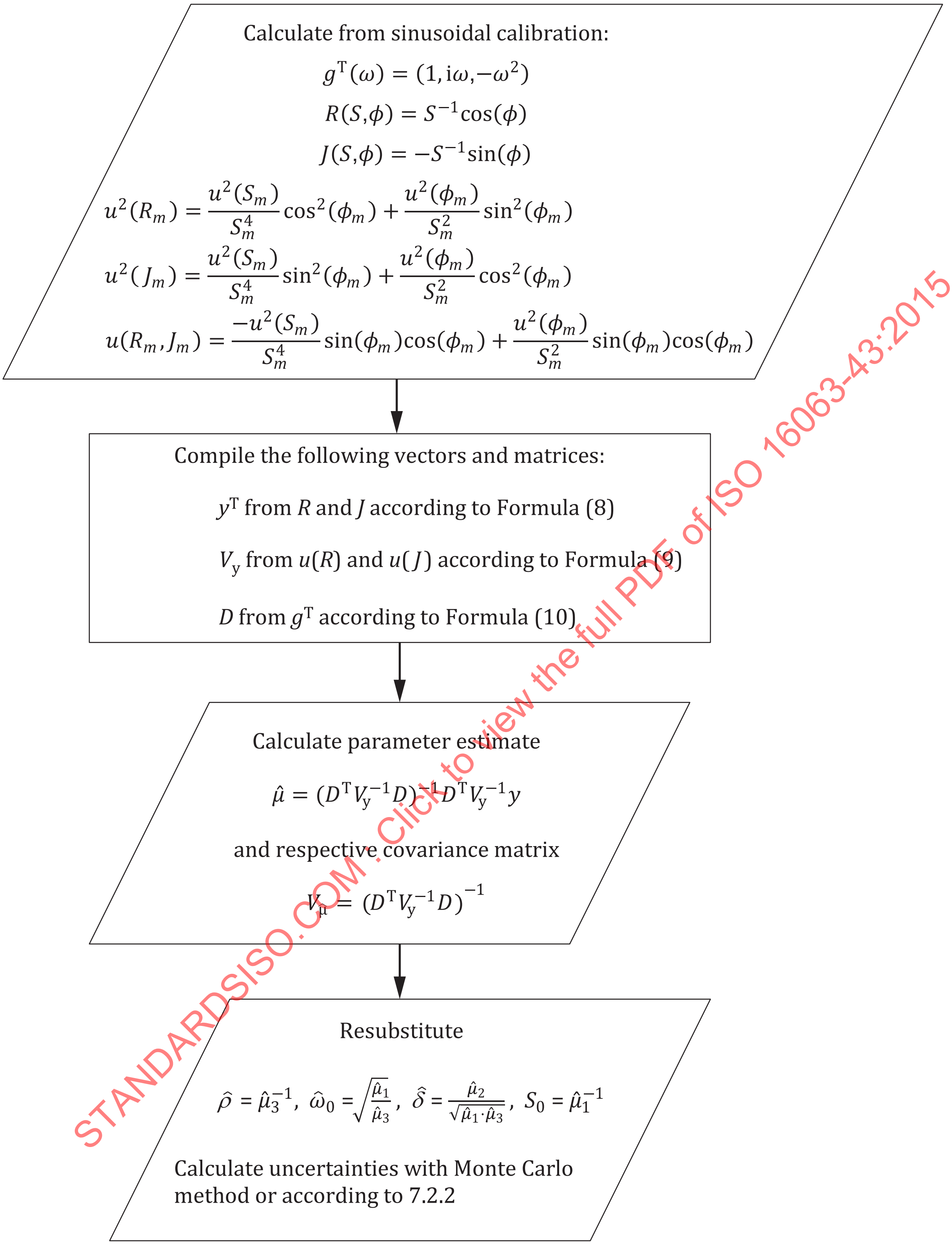


Figure 3 — Flowchart of the process of parameter identification upon sinusoidal calibration data

7.2.2 Uncertainties of model parameters by analytic propagation

In cases where the total expanded relative uncertainty of measurement of the magnitude S_m is less than 1 % and the total uncertainty of measurement of the phase ϕ_m is less than 2°, a conventional propagation of uncertainty is feasible, although Formula (13) states a strongly nonlinear relationship.

With the transformation matrix given in Formula (15):

$$A_{\mu} = \begin{pmatrix} \frac{\partial \rho}{\partial \mu_1} & \frac{\partial \rho}{\partial \mu_2} & \frac{\partial \rho}{\partial \mu_3} \\ \frac{\partial \omega_0}{\partial \mu_1} & \frac{\partial \omega_0}{\partial \mu_2} & \frac{\partial \omega_0}{\partial \mu_3} \\ \frac{\partial \delta}{\partial \mu_1} & \frac{\partial \delta}{\partial \mu_2} & \frac{\partial \delta}{\partial \mu_3} \end{pmatrix} \quad (15)$$

the covariance matrix of the model parameters $V_{\rho,\omega_0,\delta}$ can be calculated from the covariance matrix $V_{\hat{\mu}}$ by Formula (16):

$$V_{\rho,\omega_0,\delta} = A_{\mu} V_{\hat{\mu}} A_{\mu}^{\mathrm{T}} \quad (16)$$

where the square roots of the diagonal elements of $V_{\rho,\omega_0,\delta}$ state the uncertainties of the model parameters.

The uncertainty for S_0 can be calculated accordingly by substituting S_0 for ρ in Formula (15).

NOTE The nonlinear relationship in Formulae (13) and (14) requires an appropriate handling of the uncertainty propagation in accordance with ISO/IEC Guide 98-3/Supplement 1 for the general case. Only in the case of reduced input uncertainties is the linearization described in 7.2.2 applicable. For the given model, a comprehensive description of the general case is given in Reference [1].

7.3 Identification by shock calibration data in the frequency domain

7.3.1 Identification of the model parameters

Starting from calibration measurements with shock excitation in accordance with, for example, ISO 16063-13 or ISO 16063-22, it is possible to estimate the model parameters using a special pre-processing step with subsequent identification similar to the procedure described in the previous clause.

For the substitution, a discretization of the continuous time given in Formula (1) is necessary. For that purpose, the classical s-transform is used, which leads in the case of Formula (1) to the transformed equation given in Formula (17):

$$\left(s^2 + 2\delta\omega_0 s + \omega_0^2\right) X\left(s\right) = \rho\, A\left(s\right) \quad (17)$$

The discretization follows, for example, a bilinear mapping of the s-plane to the z-plane of the kind given in Formula (18):

$$s \rightarrow \frac{2}{T} \cdot \frac{1 - z^{-1}}{1 + z^{-1}} \quad (18)$$

where T is the sampling interval.

NOTE 1 It is reported in Reference [10] that a systematic deviation between continuous frequency representation and discretized frequencies due to the transform is reduced by applying a pre-warping.

Substituting Formula (18) into Formula (17), taking the sampled time series x_k and a_k for the respective variables and applying the back shift operator z^{-1} of the z-transform properly ($z^{-1} \cdot x_k = x_{k-1}$), results in the discretized version of the model equation (see Reference [3]), as given in Formula (19):

$$x_k = -c_1 x_{k-1} - c_2 x_{k-2} + b\left(a_k + 2a_{k-1} + a_{k-2}\right) \qquad (19)$$

NOTE 2 The error introduced by the discretization with respect to the sample rate needs some further investigation and consideration in the uncertainty budget. At the time of writing, no publications are available on this topic.

The parameters of Formula (19) are related to the continuous model parameters according to Formula (20):

$$\begin{aligned} b &= \frac{\rho T^2}{4\Lambda} \\ c_1 &= \frac{\omega_0^2 T^2 - 4}{2\Lambda} \\ c_2 &= \frac{4 - 4\delta\omega_0 T + \omega_0^2 T^2}{4\Lambda} \end{aligned} \qquad (20)$$

where $\Lambda = 1 + \delta\omega_0 T + \dfrac{\omega_0^2 T^2}{4}$

It is clear from this derivation that the parameters of the discretized model are dependent upon the sample rate T^{-1} and are therefore closely related to the calibration set-up. The sample rate used for the measurement should be at least five times greater than the frequency at which the first significant resonance of the transducer under calibration occurs. In order to avoid additional significant uncertainty components due to lack of resolution over frequency regions in which resonances occur, a better sampling frequency would be a factor of 10 or more greater than the frequency at which the first significant resonance of the transducer under calibration occurs.

The introduced discrete time model Formula (19) has a (periodic) frequency response of the form given in Formula (21):

$$H\left(e^{i\Omega}\right) = \frac{b\left(1 + 2e^{-i\Omega} + e^{-i2\Omega}\right)}{1 + c_1 e^{-i\Omega} + c_2 e^{-i2\Omega}} \qquad (21)$$

where $\Omega = \omega / f_s = \omega T$ is the circular frequency normalized to the sample rate.

Just like Formula (5), the inverse of this frequency response $G\left(e^{i\Omega}\right)$ is linear in the parameters, after some obvious substitution, given in Formula (22):

$$G\left(e^{i\Omega}\right) = H^{-1}\left(e^{i\Omega}\right) = \frac{1 + c_1 e^{-i\Omega} + c_2 e^{-i2\Omega}}{b\left(1 + 2e^{-i\Omega} + e^{-i2\Omega}\right)} = \frac{v_1 + v_2 e^{-i\Omega} + v_3 e^{-i2\Omega}}{1 + 2e^{-i\Omega} + e^{-i2\Omega}} \qquad (22)$$

with the substitute vector, given in Formula (23):

$$v^{T} = \left[v_1, v_2, v_3\right] = \left[1/b, c_1/b, c_2/b\right] \qquad (23)$$

With this inverse frequency response the general approach taken already in 7.2 can be followed. For the sake of completeness this is worked out in more detail as follows.

Let X_n and A_n be the components of the discrete Fourier transform (DFT) of the sampled time series x_k and a_k respectively with $n = 0, 1, ..., N - 1$. Here, the influence of a conditioning amplifier can be eliminated by multiplying A_n by the measured complex frequency response of the amplifier for the frequency $\frac{1}{2T}\frac{n}{N}$ in order to compensate the response later in Formula (24). In cases where AC-coupled conditioning amplifiers are used, the terms for $n = 0$ should be omitted, because they describe the DC component of the signals, which vanishes for X_0. Formula (22) implies the relation given in Formula (24):

$$G_n = \frac{A_n}{X_n} = f_n^{\mathrm{T}} v \tag{24}$$

with the vector f_n^{T} given as in Formula (25) from Formula (22):

$$f_n^{\mathrm{T}} = \left[1, \mathrm{e}^{-\mathrm{i}(2\pi/N)n}, \mathrm{e}^{-\mathrm{i}2(2\pi/N)n}\right] \cdot \frac{1}{1 + 2\mathrm{e}^{-\mathrm{i}(2\pi/N)n} + \mathrm{e}^{-\mathrm{i}2(2\pi/N)n}} \tag{25}$$

Formula (24) is the analogue to Formula (5). Note that the careful choice of the values for n offers the opportunity to limit the process to the relevant frequency range of the measurement.

Estimation of the parameter vector v is then performed by weighted least square fitting, i.e. by minimizing Formula (26):

$$\chi^2 = \sum_{n \in v} \left(\frac{R^2\left(G_n - f_n^{\mathrm{T}} v\right)}{u^2\left[R\left(G_n\right)\right]} + \frac{J^2\left(G_n - f_n^{\mathrm{T}} v\right)}{u^2\left[J\left(G_n\right)\right]} \right) \tag{26}$$

with respect to v, where the values of n are chosen such that the relevant frequency range is covered. The weights are again chosen to be the reciprocals of the squared standard uncertainties associated with the measured G_n. This uncertainty is estimated by the relation given in Formula (27):

$$u^2\left[R\left(G_n\right)\right] = u^2\left[J\left(G_n\right)\right] = \frac{u_0^2}{\left|X_n^2\right|} \tag{27}$$

The unknown uncertainty u_0 is determined by requiring that the minimum of Formula (19) equals the degree of freedom (number of data entering the fit minus number of adjusted model parameters). The components of G_n given in Formula (28) are considered to be uncorrelated:

$$R_n = R\left(G_n\right) \text{ and } J_n = J\left(G_n\right) \quad n_1 \le n \le n_2 \tag{28}$$

NOTE 3 This assumption is treated in the appendix of Reference [2] in more detail.

The procedure for minimizing Formula (26) is similar to that described in 7.2.1, as given in Formulae (29), (30) and (31):

$$y^{\mathrm{T}} = \left(R_{n_1}, \dots, R_{n_2}, J_{n_1}, \dots, J_{n_2}\right) \tag{29}$$

$$V_{\mathrm{y}} = \begin{pmatrix} u^2\left(R_{n_1}\right) & & & & & \\ & \ddots & & & 0 & \\ & & u^2\left(R_{n_2}\right) & & & \\ & & & u^2\left(J_{n_1}\right) & & \\ & 0 & & & \ddots & \\ & & & & & u^2\left(J_{n_2}\right) \end{pmatrix} \tag{30}$$

$$D = \begin{pmatrix} R\left(f_{n_1}^{\mathrm{T}}\right) \\ \vdots \\ R\left(f_{n_2}^{\mathrm{T}}\right) \\ J\left(f_{n_1}^{\mathrm{T}}\right) \\ \vdots \\ J\left(f_{n_2}^{\mathrm{T}}\right) \end{pmatrix} \tag{31}$$

This means the weighted least square estimate of the parameters can be calculated according to Formula (32):

$$\hat{v} = \left(D^{\mathrm{T}} V_{\mathrm{y}}^{-1} D\right)^{-1} D^{\mathrm{T}} V_{\mathrm{y}}^{-1} y \tag{32}$$

The uncertainties associated with the estimated parameter set $\hat{v}$; i.e. the covariance matrix, are given by Formula (33):

$$V_{\hat{v}} = \left(D^{\mathrm{T}} V_{\mathrm{y}}^{-1} D\right)^{-1} \tag{33}$$

The estimate of the original (continuous time model) parameter set can subsequently be calculated via Formula (23) and Formula (32) according to Formula (34):

$$\begin{aligned} \rho &= \frac{16}{T^2\left(v_1 - v_2 + v_3\right)} \\ \omega_0^2 &= \frac{4\left(v_1 + v_2 + v_3\right)}{T^2\left(v_1 - v_2 + v_3\right)} \\ \delta &= \frac{v_1 - v_3}{\sqrt{v_1 - v_2 + v_3}\sqrt{v_1 + v_2 + v_3}} \\ S_0 &= \frac{2}{\sqrt{v_1 + v_2 + v_3}} \end{aligned} \tag{34}$$

Figure 4 gives a flowchart representation of the whole analysis process.

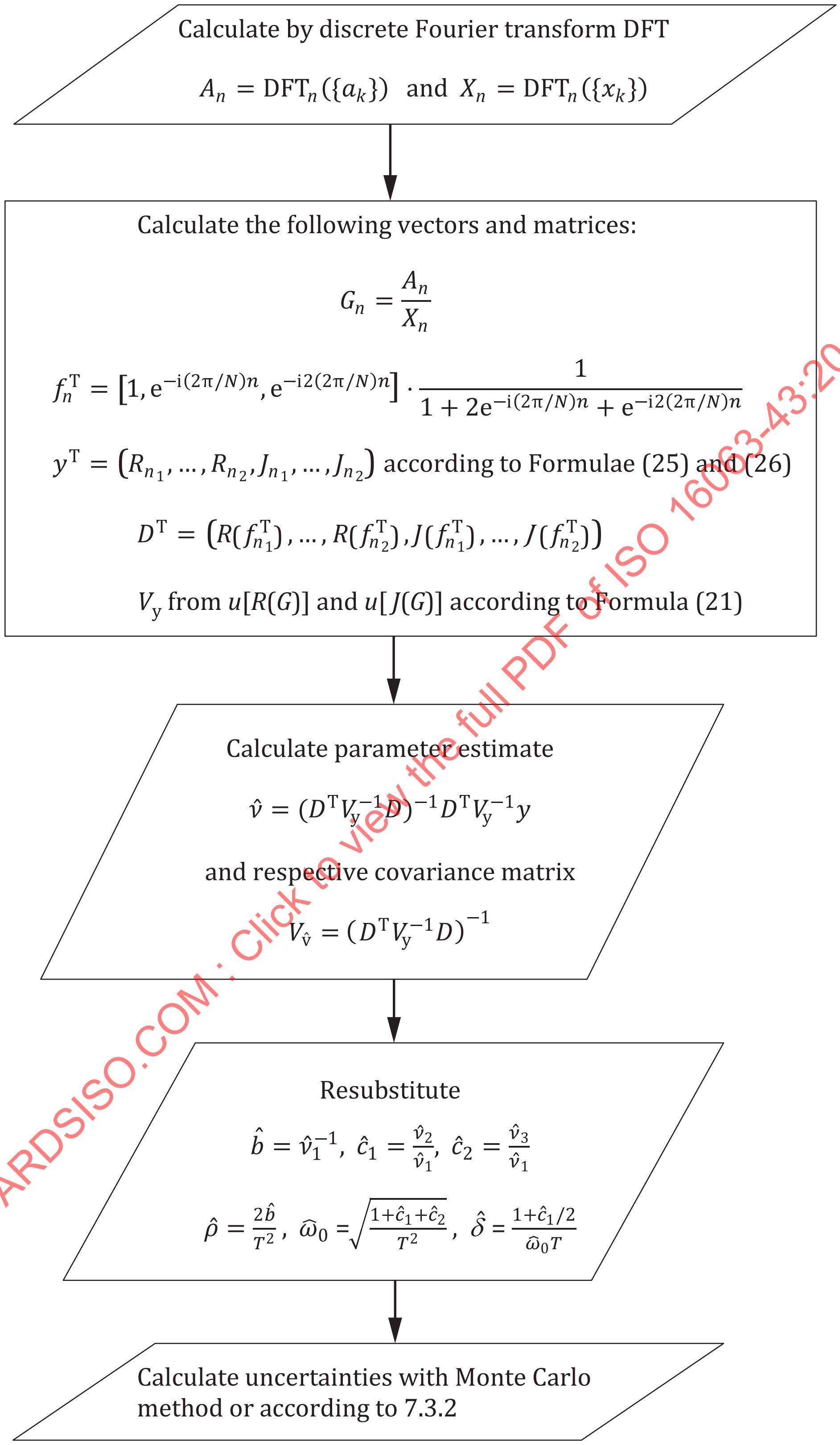


Figure 4 — Flowchart of the process of parameter identification upon shock calibration data in the frequency domain

7.3.2 Uncertainties of model parameters by analytical propagation

In cases where the total expanded relative uncertainty of measurement for the sampled time series is sufficiently small, a conventional propagation of uncertainty is feasible, although Formula (34) states a strongly nonlinear relationship.

The covariance matrix $V_{\hat{v}}$ of the identified parameters is calculated according to Formula (33).

With the transformation matrix given in Formula (35):

$$A_v = \begin{pmatrix} \frac{\partial \rho}{\partial v_1} & \frac{\partial \rho}{\partial v_2} & \frac{\partial \rho}{\partial v_3} \\ \frac{\partial \omega_0}{\partial v_1} & \frac{\partial \omega_0}{\partial v_2} & \frac{\partial \omega_0}{\partial v_3} \\ \frac{\partial \delta}{\partial v_1} & \frac{\partial \delta}{\partial v_2} & \frac{\partial \delta}{\partial v_3} \end{pmatrix} \quad (35)$$

the covariance matrix of the model parameters $V_{\rho,\omega_0,\delta}$ can be calculated from the covariance matrix $V_{\hat{v}}$, as given in Formula (36):

$$V_{\rho,\omega_0,\delta} = A_v V_{\hat{v}} A_v^T \quad (36)$$

where the square roots of the diagonal elements of $V_{\rho,\omega_0,\delta}$ state the uncertainties of the model parameters.

The uncertainty for S_0 can be calculated accordingly by substituting S_0 for ρ in Formula (35).

NOTE The nonlinear relationship in Formula (34) again requires an appropriate handling of the uncertainty propagation in accordance with ISO/IEC Guide 98-3/Supplement 1 for the general case. The linearization described in 7.3.2 is only applicable in the case of reduced input uncertainties. For the given model, a comprehensive description of the general case is given in Reference [3].

8 Practical considerations

8.1 The influence of the measurement chain

The physical model, which gave guidance for the model in Formula (1), is that of a simple seismic transducer. As such, it does not allow for different frequency responses of the additional components in the measurement chain, e.g. a charge amplifier. In order to account for the conditioning elements of a measuring chain, it is necessary to either compensate the frequency response of the respective component in the measurement channel or to introduce its response in the acceleration measuring channel by appropriate filtering.

The goal is symmetry in the frequency response of the acceleration measuring channel and the conditioning part of the channel of the device under test. The compensation is necessary for magnitude as well as for the phase shift component of the compensated response.

NOTE A third approach would be an extension of the model that incorporates the conditioning amplifier, thus it would be possible to perform a parameter identification of the measuring chain. However, such an approach has not been investigated or even validated at the time of writing.